

# Subject: - Structural Design-I

Sem - 4<sup>th</sup>

Course - Diploma

Branch - Civil

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Chapters - 1

Working Stress Method

## Objective of design :-

- (i) To decide the size (dimensions) of the members and the amount of reinforcement required.
- (ii) To check whether the adopted section will perform safely and satisfactorily during the life time of the structure.

## Different methods of design of concrete structures :-

- (i) Working stress Method
- (ii) Ultimate load Method
- (iii) Limit state Method

## Working stress Method :-

- (i) This method of design was the oldest one and based on the elastic theory and assumes that both steel and concrete are elastic and obeys Hooke's law.
- (ii) Stress is directly proportional to strain upto the point of collapse.
- (iii) The basis of this method is that permissible stresses are not exceeded anywhere in the structure when it is subjected to worst combination of working loads.

(iv) In this method, the ultimate stresses of concrete and yield strength or 0.2% Proof stress of steel are divided by factors of safety to obtain permissible stresses.

### (\*) Ultimate Load Method :-

(i) Ultimate load or collapse load is used as design load. Ultimate load is obtained by multiplying working load with the load factor.

(ii) The load factor gives exact margin of safety in terms of load.

(iii) It uses stress-strain curve and takes plastic behaviour of material into account.

### (\*) Limit State Method :-

(i) The acceptable limit of safety and serviceability requirements before failure occurs is called limit state and this method is called as limit state method.

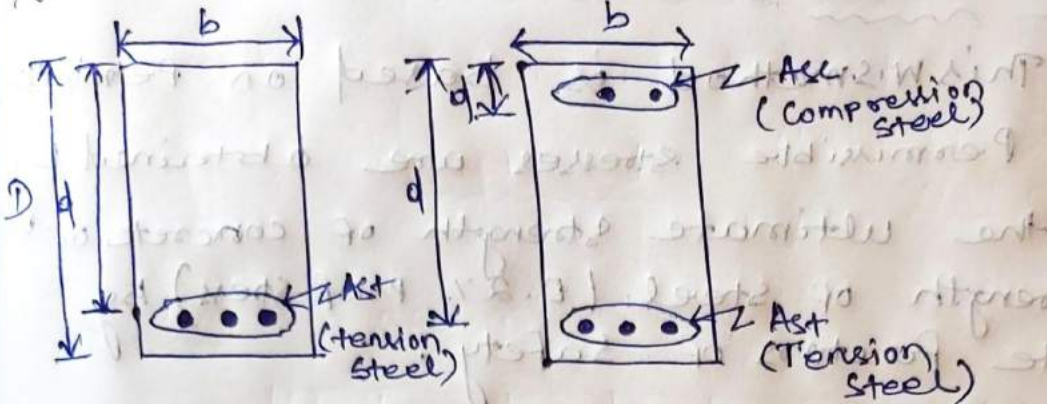
(ii) This method is based on the concept of safety at ultimate loads (Ultimate load method) and serviceability at working loads (working stress method).

(iii) There are following two types of limit state methods :-

(a) Limit state of collapse

(b) Limit state of serviceability

Types of R.C. sections :-



(Single reinforced)

(Doubly reinforced)

[Steel reinforcement provided only in tension zone]

[Steel reinforcement provided both in compression & steel zone]

Assumptions in W.S.M

Assumptions are drawn on the basis of working stress method :-

- ① A section which is plane before bending remains plane after bending.
- ② The concrete and steel reinforcement are perfectly bonded.
- ③ All tensile stresses are taken up by steel and none by concrete.
- ④ The stress-strain relationship of steel and concrete under working loads is a straight line.
- ⑤ The modulus of elasticity of steel ( $E_s$ ) and concrete ( $E_c$ ) are constant.
- ⑥ There are no initial stresses in steel and concrete.

## Permissible stresses:

This is a method, is based on permissible stresses. Permissible stresses are obtained by dividing the ultimate strength of concrete or yield strength of steel (0.2% proof stress) by appropriate factors of safety.

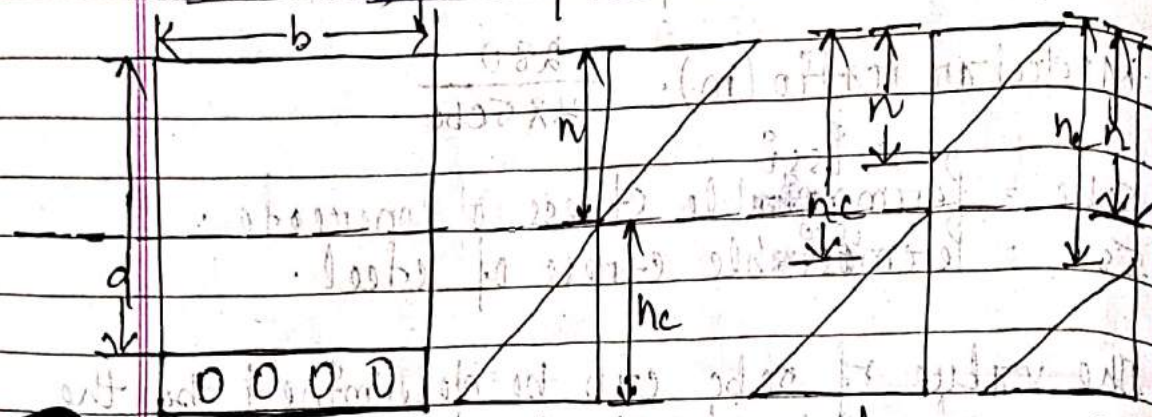
factors of safety are

(i) for concrete

- (a) in bending compression - 3.0
- (b) in direct compression - 4.0

(ii) for steel - 1.78

→ Concepts of balanced, under reinforced & over reinforced sections:-



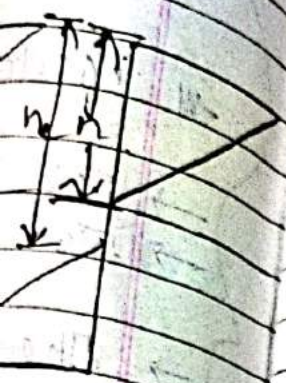
balanced section  $n = n_c$   
 under reinforced section  $n < n_c$   
 over reinforced section  $n > n_c$

Q: M.O.R of a R.C.C beam 300mm wide & 500mm effective depth is reinforced with 3 bars of 16mm. M20 concrete & Fe 415 steel is used.

Ans: Given data  
 $b = 300\text{mm}$   
 $d = 500\text{mm}$   
 Grade of concrete = M20  
 Grade of steel = Fe 415  
 $\sigma_{cbc} = 7\text{ N/mm}^2$   
 $\sigma_{st} = 230\text{ N/mm}^2$

Step-1

$$m = \frac{280}{3 \times \sigma_{cbc}} = \frac{280}{3 \times 7}$$



overly reinforced section  
 $n > n_c$   
500 mm  
base of  
fe used.

$$= 0.0019 \frac{280}{21}$$

$$= 13.88$$

step-2

$$\Rightarrow A_{st} = \text{No. of bars} \times \frac{\pi}{4} \times (d)^2$$

$$= 3 \times \frac{\pi}{4} \times (16)^2$$

$$= 3 \times \frac{\pi}{4} \times 256$$

$$= 603.18 \text{ mm}^2$$

step-3

$$\Rightarrow n_c = \frac{m \sigma_{sc}}{\sigma_{st}} = \frac{n_c}{d - n_c}$$

$$\Rightarrow \frac{13.88 \times 7}{230} = \frac{n_c}{500 - n_c}$$

$$\Rightarrow 0.405 = \frac{n_c}{500 - n_c}$$

$$\Rightarrow 0.405(500 - n_c) = n_c$$

$$\Rightarrow 202.5 - 0.405 n_c = n_c$$

$$\Rightarrow 202.5 = n_c + 0.405 n_c$$

$$\Rightarrow 202.5 = n_c (1 + 0.405)$$

$$\Rightarrow 202.5 = n_c (1.405)$$

$$\Rightarrow n_c = \frac{202.5}{1.405} = 144.12 \text{ mm}$$

Step-4

$$\Rightarrow n = \frac{b n^2}{2} = M A s t (d - n)$$

$$\Rightarrow 300 \times \frac{n^2}{2} = 13.33 \times 603.18 (500 - n)$$

$$\Rightarrow \frac{300 n^2}{2} = 13.33 \times 301590 - 603.18 n$$

$$\Rightarrow \frac{n^2}{2} = \frac{13.33 \times 301590 - 603.18 n}{300}$$

$$\Rightarrow \frac{n^2}{2} = \frac{26800 \times 603.18 (500 - n)}{300}$$

$$\Rightarrow \frac{n^2}{2} = 26.80 (500 - n)$$

$$\Rightarrow n^2 = 2 \times 26.80 (500 - n)$$

$$\Rightarrow n^2 = 53.6 (500 - n) \leftarrow$$

$$\Rightarrow n^2 = 26800 - 53.6 n \leftarrow$$

$$\Rightarrow n^2 + (53.6)n - 26800 = 0$$

Formulae

$$n = \frac{-b \pm (\sqrt{b^2 - 4ac})}{2a}$$

$$\Rightarrow n = \frac{-(53.6) \pm \sqrt{(53.6)^2 - 4 \times 1 \times (-26800)}}{2}$$

$$\Rightarrow n = 139.13 \text{ mm}$$

Step - 5

under-reinforced section ( $n < n_c$ )

M.O.R.

$$M_u = \sigma_{st} A_{st} (d - n/3)$$

$$= 230 \times 603.18 (500 - 139.13/3)$$

$$= 138731.4 (500 - 46.37)$$

$$= 138731.4 \times 453.63$$

$$= 62956840.11 \text{ N/mm}$$

$$= 62.95 \text{ kNm}$$

~~over-reinforced section ( $n > n_c$ )~~

~~M.O.R. =  $\sigma_{st} A_{st} (d - n/3)$~~

Type of steel:-

There are 2 types of steels:-

(i) Mild steel.

(ii) High yield strength deformed steel.

- (i) Mild steel:-  $f_e 250, f_e 215$
- (ii) High yield strength deformed steel:-  $f_e 415, f_e 500, f_e 550$



Q: Find the M.O.R of a RCC cantilever beam of 300mm width & 500mm effective depth reinforced with 2 bars of 16mm diameter. Use M20 concrete & Fe 415 steel.

Sol:-  $\sigma_{cbc} = 7 \text{ N/mm}^2$  grade  
 $\sigma_{st} = 230 \text{ N/mm}^2$  M20 (concrete)  
Fe 415 (steel)

Step-1  $\Rightarrow m = \frac{280}{3 \times \sigma_{cbc}}$   
 $= \frac{280}{3 \times 7}$   
 $= \frac{280}{21}$   
 $= 13.33$

Step-2  $\Rightarrow A_{st} = \text{no. of bars} \times \frac{\pi}{4} \times (d)^2$   
 $= 2 \times \frac{\pi}{4} \times (16)^2$   
 $= 2 \times \frac{\pi}{4} \times 256$   
 $= 402.12$

Step-3  $\Rightarrow \frac{m \sigma_{cbc}}{\sigma_{st}} = \frac{nc}{d - nc}$   
 $\frac{13.33 \times 7}{230} = \frac{nc}{500 - nc}$   
 $\Rightarrow 0.405 = \frac{nc}{500 - nc}$   
 $\Rightarrow 0.405 (500 - nc) = nc$

beam of length 1 meter. Use

create reel

$$\Rightarrow 202.5 - 0.405nc = nc$$

$$\Rightarrow 202.5 = nc + 0.405nc$$

$$\Rightarrow 202.5 = nc(1 + 0.405)$$

$$\Rightarrow 202.5 = nc(1.405)$$

$$\Rightarrow nc = \frac{202.5}{1.405}$$

$$\Rightarrow nc = 144.12 \text{ mm}$$

Step-4

$$\Rightarrow n \Rightarrow \frac{b^2}{2} = m \cdot A \pm (d - n)$$

$$\Rightarrow 300 \frac{n^2}{2} = 13.33 \times 402.12 (500 - n)$$

$$\Rightarrow 300 \frac{n^2}{2} = 5360.25 (500 - n)$$

$$\Rightarrow \frac{n^2}{2} = \frac{5360.25 (500 - n)}{300}$$

$$\Rightarrow \frac{n^2}{2} = 17.86 (500 - n)$$

$$\Rightarrow n^2 = 2 \times 17.86 (500 - n)$$

$$\Rightarrow n^2 = 17860 - 35.72n$$

$$\Rightarrow n^2 + (35.72)n - 17860 = 0$$

$$= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\Rightarrow n = \frac{-35.72 \pm \sqrt{(35.72)^2 - 4 \cdot 1 \cdot (-17860)}}{2 \cdot 1}$$

$$= \frac{-35.72 \pm \sqrt{1215.91 - (-71440)}}{2}$$

$$= 116.96$$

Step-5 M.O.R

$$\Rightarrow M_R = \sigma_{st} A_{st} (d - n/3)$$

$$= 230 \times 402.12 \left( 500 - \frac{116.96}{3} \right)$$

$$= 92487.6 (500 - 38.19)$$

$$= 426377.08 \cdot 48$$

Q: Find the M.O.R of a RCC cantilever beam of 300mm width & 600mm effective depth reinforced with 2 bars of 16mm diameter. The bending moment of beam is 150 kNm. Also find out stress in steel ( $\sigma_{st}$ ) & concrete ( $\sigma_{cbc}$ ).

Sol:-

$d = 500 \text{ mm}$   
 $b = 300 \text{ mm}$

$n = 116.96$

$BM = 150 \text{ kNm}$

$$= 150 \times 1000 = 1000$$

$$= 150 \times 10^3 = 10^3$$

$$= 150 \times 10^6$$

$$\Rightarrow M_R = \sigma_{st} A_{st} (d - n/3)$$

$$= \sigma_{st} \cdot 402 \cdot 12 \left( 500 - \frac{116.96}{3} \right)$$

$$= 185382 \cdot 68 \cdot \sigma_{st}$$

~~$M = 180 \times 10^6$~~

$$\Rightarrow M = M_{st}$$

$$\Rightarrow M = 180 \times 10^6 = 185382 \cdot 68 \cdot \sigma_{st}$$

$$\Rightarrow \sigma_{st} = \frac{180 \times 10^6}{185382 \cdot 68}$$

$$\Rightarrow \sigma_{st} = 809.13$$

$$\Rightarrow \sigma_{cbc} (\sigma_c) = \frac{\sigma_c}{n} = \frac{\sigma_{st} / m_s}{d - n}$$

$$\Rightarrow \frac{\sigma_c}{116.96} = \frac{809.13 / 13.33}{500 - 116.96}$$

$$\Rightarrow \frac{\sigma_c}{116.96} = \frac{60.69}{383.04}$$

$$\Rightarrow \frac{\sigma_c}{116.96} = 0.158$$

$$\Rightarrow \sigma_c = 0.158 \times 116.96$$

$$\Rightarrow \sigma_c = 18.49$$

## Chapter - 2

# Limit State Method (LSM)

Date 23/07/20  
Page 10

Def<sup>n</sup> :-

In this method the structure shall be design to with stand safety of all loads liable (responsible) to act on in through out its life.

Therefore some limitation are drawn on deflection & cracks. The acceptable limit for the service & safety requirements. Before failure occur that is called as limit state. Then the method is used for limit state is called as limit state method.

Objective :-

The objective of limit state design is based on the concept of achieving an acceptable probability that a structure will not become non serviceable in its life time for the use for which it is intended (purpose).

Concept of limit state :-

- > The structure shall be design to with stand safety of all loads liable to act on in through out its life.
- > Some limitation are drawn on deflection & cracks.
- > The acceptable limit for the service & safety requirements.
- > Before failure occur that is called as limit state.
- > The the method is used for limit state is called as limit state Method (LSM).
- > The objective of limit state design is based on the concept of achieving an acceptable

probability - increases  
- A structure will not become non-serviceable in its life time for the use for which it is intended (purpose).

Types of Limit state:-

- (1) Limit state of safety / collapse.
- (2) Limit state of serviceability.

(1) Limit state of collapse:-

It is also called as strength limit state as it corresponds to the max<sup>m</sup> load carrying capacity. That means the safety requirements of the structure. The limit state of collapse is reached from the collapse of the whole or part of the structure. The limit state of collapse in design are considered as follows:-

- (i) Limit state of collapse in flexure (bending).
- (ii) " " " " compression.
- (iii) " " " " shear.
- (iv) " " " " torsion (twisting).

(2) Limit state of serviceability:-

A structure is of no use as if it is not serviceable. Thus, this limit state is introduced to prevent excessive deflection & cracking.

Limit state of serviceability are considered as follows:-

- (i) Limit state of deflection.
- (ii) " " " " Cracking.

### (iii) Limit state of vibration

DT → 27/01/20

#### → Advantages of LSM over WSM :-

(i) LSM is based on the actual stress-strain curve for steel & concrete. For concrete, the stress-strain curve is non-linear, whereas WSM is based on the elastic theory which assumes both steel & concrete are elastic & the stress-strain curve is linear for both.

(ii) In LSM partial safety factors are applied to get the design value of stresses. Like WSM in working state method, the factors of safety are applied to the yield stress to get the permissible stress.

(iii) In LSM design loads are obtained or calculated by multiplying partial safety factors of load to the working loads. Whereas, no safety factor is used for loads in WSM.

(iv) In LSM exact margin of safety is none. However, in WSM exact margin of safety is none.

(v) LSM is more economical as it gives thinner sections whereas, WSM gives thicker sections. Therefore, it is less economical.

#### → Characteristic strength of material :-

$$C.S. = \text{Mean strength} - 1.65 \sigma$$

where,

$s$  = Standard deviation.

The value of  $k = 1.64$  is per corresponding to 5% probability.

→ Characteristic strength of concrete —

The term characteristic strength means that value of strength of material below which not more than 5% of the test results are expected to form. It is denoted by  $f_{ck}$ . The unit is  $N/mm^2$ .

→ Characteristic strength of steel —

→ It is taken as the max<sup>m</sup> yield stress or 0.2% prove stress specified by various IS (Indian standards).

→ The case of mild steel is taken as equal to min<sup>m</sup> yield strength & in case of HPS (High yield strength / deformed steel) it is taken as equal to 0.2% prove stress.

→ Characteristic strength of loads —

$C.S = \text{Mean strength} + k \cdot s$ .

It is the ultimate load that liable to come on structure during its life span.

Recommended

IS code may be used for this purpose —

IS 875 (part-1) it is used for dead loads

IS 875 (part-2) it is used for imposed loads (live loads).

IS 875 (part-3) it is used for wind loads.



IS 875 (part - 1) if it is used for snow load.  
IS 875 (part - 4) if it is used for earthquake load.

→ Partial safety of factors:

(i) The value of partial safety of factors for limit state of collapse should be taken as follows:-

- In case of the concrete the value of partial safety of factors is 1.5.
- In case of the steel the value of partial safety of factors is 1.15.

(ii) The value of partial safety of factors for limit state of serviceability should be taken as follows:-

- It will vary bet<sup>n</sup> 0.8 to 1.0.

No.	Combination	LS of collapse			LS of serviceability		
		DL	LL	WL	DL	LL	WL
01	DL + LL	1.5	1.5	N/A	1.0	1.0	N/A
02	DL + WL	1.5	N/A	1.5	1.0	N/A	1.0
03	DL + LL + WL	1.2	1.2	1.2	1.0	0.8	0.8

→ Dead loads:

The load which acts on a structure constantly. This type of loads are - the permanent loads which are always present.

→ Live loads:

The loads keep on changing from time to time. They are also called as imposed load.

# Chapter - 03

## Analysis & Design of single & Double Reinforced sections (Lem)

Date 28/01/21  
Page 15

1st

→ Limit state of collapse in flexure (bending)  
Assumptions in limit state of collapse in flexure :-

-i Plain sections normal to the axis remain plain after bending. It means that the strain at any pt. in the cross-section is proportional to the distance from the neutral axis.

-ii "The max<sup>m</sup> strain in concrete at the outer most compression fiber is taken as 0.0035 in bending."

-iii The relationship bet<sup>n</sup> the stress-strain distribution in concrete is assume to be parabolic. For design purpose the compressive strain of concrete is assume to be 0.67 times of the characteristic strength of concrete. The partial safety factor  $\gamma_{mc} = 1.5$  shall be applied as follows.

max<sup>m</sup> compressive stress in concrete

$$\sigma_c = \frac{0.67 f_{ck}}{1.5} \text{ (partial safety factor)}$$

where,  $f_{ck}$  = characteristic strength of concrete.

- iv The tensile strength of concrete is ignored.
- v The stresses in the reinforcement are taken from the stress-strain curve for the type of steel used. For the design purpose the partial safety factor  $\gamma_{ms} = 1.15$  shall be applied.
- vi The max<sup>m</sup> strain in tension reinforcement is

The section of failure shall not be less than:

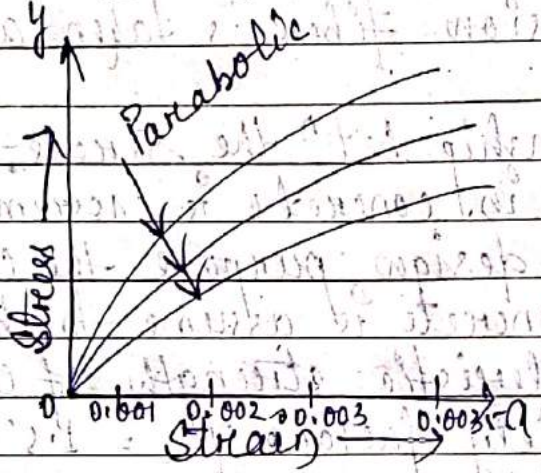
$$\frac{f_y}{1.5 E_s} + 0.002$$

where;

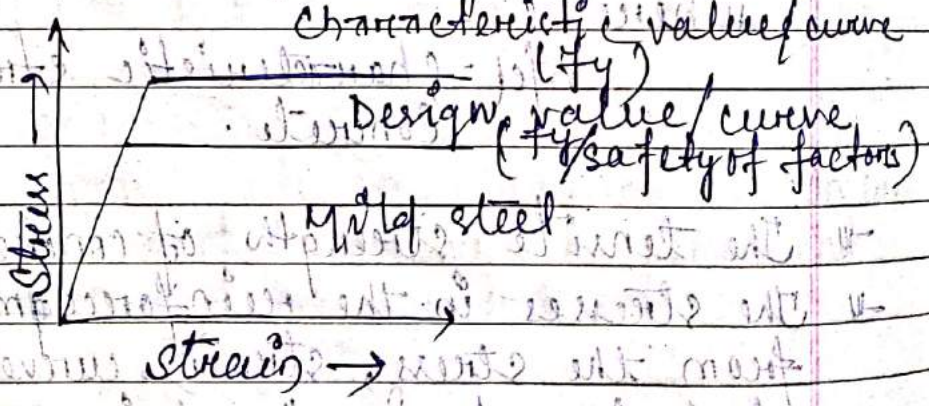
$f_y$  = characteristic strength of steel.

$E_s$  = Modulus of elasticity of steel.

Q.11 → Stress-strain relationship bet<sup>n</sup> concrete:-



Q.12 → Stress-strain relationship bet<sup>n</sup> steel:-



Strain →

Fig. (1)

less than

of steel.  
of steel.

concrete:-

The stress, strain, curve is shown as above in fig. (1).

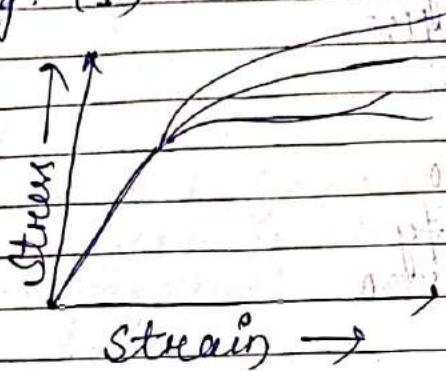


fig. (2).

- For HYSD steel is shown as above in fig. (2)
- The value of characteristic stress is taken as yield stress & the design curve is obtained after applying a factor of safety of 1.15 to yield stress.
- Then the design stress for steel is equal to  $f_y / 1.15$  or  $\frac{1}{1.15} \times f_y$

Design strength ( $F_d$ ) values of steel =  $0.87 f_y$   
 where,  
 $f_y$  = Characteristic strength of steel.

(i) For mild steel :-  
 $= f_e 250$   
 $= 0.87 f_y$   
 $= 0.87 \times 250$   
 $= 217.5$

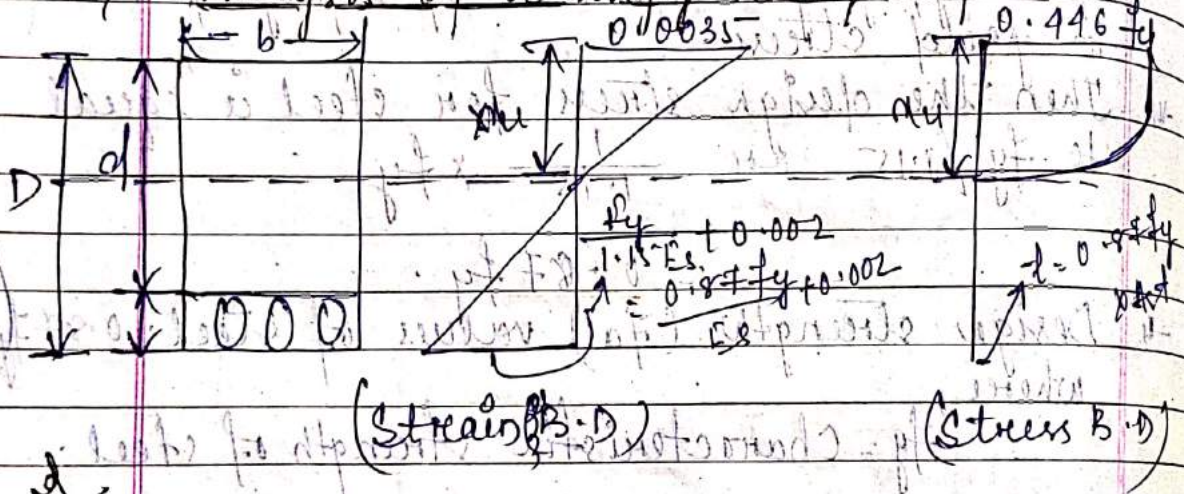
(ii) For HYSD :-

$$\begin{aligned} &\rightarrow = f_c 115 \\ &= 0.87 f_y \\ &= 0.87 \times 115 \\ &= 361.05 \end{aligned}$$

$$\begin{aligned} &\rightarrow = f_c 500 \\ &= 0.87 f_y \\ &= 0.87 \times 500 \\ &= 435 \end{aligned}$$

$$\begin{aligned} &\rightarrow = f_c 550 \quad (2) \text{ part} \\ &= 0.87 f_y \\ &= 0.87 \times 550 \\ &= 478.5 \end{aligned}$$

✓ Analysis of a singly reinforced beam



- ✓ Strain distribution
- Assumption:-
- u strain at neutral axis = 0
  - u max<sup>n</sup> or ultimate strain in concrete at extreme fiber = 0.0035
  - u strain at constant stress of  $0.67 f_c$ ,  $\frac{0.67 f_c}{E_c} = 0.002$

-w Ultimate strain in steel corresponding to max<sup>m</sup> stress at failure.

$$\epsilon_{su} = \frac{0.87 f_y}{E_s} + 0.002$$

→ Stress distribution:—

-w stress diagram is shown as above.

-w It has a parabolic shape from A & B then linear from B to C.

Assumption:—

-w Stress at neutral axis (pt. A = 0)

-w Stress at 0.002 strain (pt. B =  $\frac{0.67 f_{ck}}{1.5}$ )  
= 0.446 f<sub>ck</sub>)

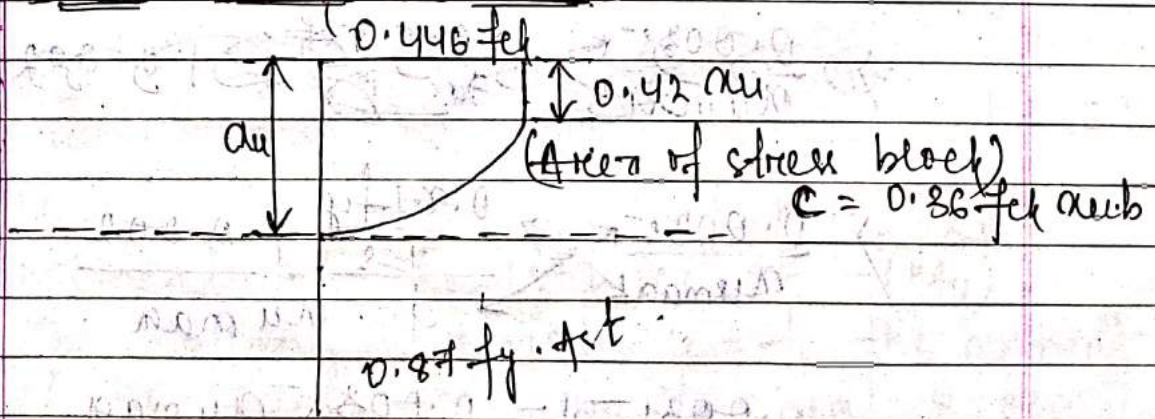
-w Stress at extreme fiber (pt. C = 0.446 f<sub>ck</sub>)

-w Below the neutral axis the concrete is assumed to be cracked max<sup>m</sup> stress in steel

$$= \frac{f_y}{1.15}$$

$$= 0.87 f_y$$

→ Stress Block Parameters:—



Total compression (C)

$$= 0.36 f_{ck} x_u \cdot b$$

Total tension (T)  
 $= 0.87 f_y \cdot A_{st}$

→ For equilibrium forces: —

$\Rightarrow T = C$   
 $\Rightarrow 0.87 f_y \cdot A_{st} = 0.36 f_{ck} \cdot m_u \cdot b$

formulae  $\Rightarrow m_u = \frac{0.87 f_y \cdot A_{st}}{0.36 f_{ck} \cdot b}$  (1)

∴ Divide "d" from both sides in equ (1)

$\Rightarrow \frac{m_u}{d} = \frac{0.87 f_y \cdot A_{st}}{0.36 f_{ck} \cdot b \cdot d}$

$\Rightarrow \frac{m_u}{d} = \frac{0.87 f_y \cdot A_{st}}{0.36 f_{ck} \cdot b \cdot d}$

formulae  $\Rightarrow \frac{m_u}{d} = \frac{0.87 f_y \cdot A_{st}}{0.36 f_{ck} \cdot b \cdot d}$  (2)

→ Limiting depth of neutral axis ( $x_{u,max}$ ): —

~~$\Rightarrow \frac{0.0031 \cdot x_{u,max}}{d - x_{u,max}} = \frac{0.87 f_y}{E_s} \cdot \frac{A_{st}}{b \cdot d}$~~

$\Rightarrow \frac{0.0031 \cdot x_{u,max}}{d - x_{u,max}} = \frac{0.87 f_y}{E_s} \cdot \frac{0.002}{d - x_{u,max}}$

$\Rightarrow 0.0031 \cdot d - 0.0031 \cdot x_{u,max} = \left( \frac{0.87 f_y}{E_s} + 0.002 \right) x_{u,max}$

$$\Rightarrow 0.0035d - 0.0035 \mu_{max} = \frac{0.87 f_y \mu_{max}}{E_s} + 0.002 \mu_{max}$$

$$\Rightarrow 0.0035d = \frac{0.87 f_y \mu_{max}}{E_s} + 0.002 \mu_{max} + 0.0035 \mu_{max}$$

$$\Rightarrow 0.0035d = \mu_{max} \left( \frac{0.87 f_y}{E_s} + 0.002 + 0.0035 \right)$$

$$\Rightarrow \frac{0.0035}{\frac{0.87 f_y}{E_s} + 0.002 + 0.0035} > \frac{\mu_{max}}{d}$$

$$\Rightarrow \frac{\mu_{max}}{d} > \frac{0.0035}{\frac{0.87 f_y}{E_s} + 0.0055}$$

here  $f_{225} = 0.53$

$f_{415} = 0.48$

$f_{500} = 0.46$

$\mu_{max} = f_{225} = 0.53d$

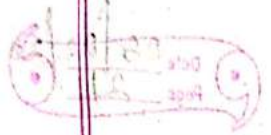
$f_{415} = 0.48d$

$f_{500} = 0.46d$

→ Moment of Resistance :- (M<sub>r</sub>)

The moment of resistance = to the moment of the couple form by to equal & opposite forces. That means to total compression (C) & total tension (T).





- Ultimate moment of resistance ( $M_u$ )

-  $M_u = C \cdot \text{lever arm} = T \cdot \text{lever arm}$

where,

$C = 0.36 f_{ck} \cdot \alpha_u \cdot b$

$T = 0.87 f_y \cdot A_{st}$

$\text{lever arm} = d - 0.42 \alpha_u$

$\Rightarrow M_u = C \cdot \text{lever arm}$

$= 0.36 f_{ck} \cdot \alpha_u \cdot b \times d - 0.42 \alpha_u$

$= 0.36 f_{ck} \cdot \alpha_u \cdot b (d - 0.42 \alpha_u)$

$= 0.36 f_{ck} \cdot \alpha_u \cdot b \cdot d (1 - 0.42 \alpha_u)$

$= 0.36 f_{ck} \frac{\alpha_u}{d} (1 - \frac{0.42 \alpha_u}{d}) b d^2$

$\Rightarrow M_u = T \cdot \text{lever arm}$

$= 0.87 f_y A_{st} \times d - 0.42 \alpha_u$

$= 0.87 f_y A_{st} (d - 0.42 \alpha_u)$

$= 0.87 f_y A_{st} \cdot d (1 - 0.42 \alpha_u)$

$= 0.87 f_y A_{st} d (1 - \frac{0.42 \times 0.87 f_y A_{st}}{0.36 f_{ck} \alpha_u b d})$

$= 0.87 f_y A_{st} d (1 - \frac{f_y A_{st}}{f_{ck} \alpha_u \cdot b d})$

$\rightarrow$  Limiting moment of resistance ( $M_{u,lim}$ ) :-

$M_{u,lim} = 0.36 f_{ck} \left( \frac{\alpha_{u,max}}{d} \right) \times \left( \frac{1 - 0.42 \alpha_{u,max}}{d} \right) b d^2$

$0.36 f_{ck} b \alpha_{u,max} (d - 0.42 \alpha_{u,max})$

mild steel = Fe 250

mulin of Fe 250

$$\frac{\alpha_{max}}{d} = 0.53$$

mulin of (Fe 250) =  $0.36 f_{ck} \times 0.53$

$$= 1 - (0.42 \times 0.53) b d^2$$

$$= 0.148 f_{ck} b d^2$$

→ HYSD - Fe 415

mulin of Fe 415

for Fe 415

$$\frac{\alpha_{max}}{d} = 0.48$$

mulin (Fe 415) =  $0.36 f_{ck} \frac{\alpha_{max}}{d} \cdot 1 - 0.42 \frac{\alpha_{max}}{d} b d^2$

$$= 0.36 f_{ck} \times 0.48$$

$$\cdot (1 - 0.42 \times 0.48) b d^2$$

$$= 0.138 f_{ck} b d^2$$

→ HYSD - Fe 500

for Fe 500

$$\frac{\alpha_{max}}{d} = 0.46$$

mulin (Fe 500) =  $0.36 f_{ck} \frac{\alpha_{max}}{d} (1 - 0.42 \alpha_{max}) b d^2$

$$= 0.86 f_{ck} \times 0.46 (1 - 42 \times 0.46) b d^2$$

$$= \boxed{0.133 f_{ck} b d^2}$$

for M15 grade of concrete & Fe 250 steel.

$$m_{u\ lim} = 0.148 f_{ck} b d^2$$

$f_{ck}$  of M15 = 15 N/mm<sup>2</sup>

$$\Rightarrow m_{u\ lim} = 0.148 f_{ck} b d^2$$

$$= 0.148 \times 15 b d^2$$

$$= \boxed{2.22 b d^2}$$

for M20 grade of concrete & Fe 250 steel.

$$m_{u\ lim} = 0.148 f_{ck} b d^2$$

$f_{ck}$  of M20 = 20 N/mm<sup>2</sup>

$$\Rightarrow m_{u\ lim} = 0.148 f_{ck} b d^2$$

$$= 0.148 \times 20 b d^2$$

$$= \boxed{2.96 b d^2}$$

for M25 grade of concrete & Fe 250 steel.

$$m_{u\ lim} = 0.148 f_{ck} b d^2$$

$f_{ck}$  of M25 = 25 N/mm<sup>2</sup>

$$\Rightarrow m_{u\ lim} = 0.148 f_{ck} b d^2$$

$$= 0.148 \times 25 b d^2$$

$$= 3.70 b d^2$$

for M30 grade of concrete & Fe250 steel.

$$m_{ulim} = 0.148 f_{ck} b d^2$$

$$f_{ck} \text{ of M30} = 30 \text{ N/mm}^2$$

$$\Rightarrow m_{ulim} = 0.148 f_{ck} b d^2$$

$$= 0.148 \times 30 b d^2$$

$$= 4.44 b d^2$$

$E_s$ (modulus of elasticity) $= 2 \times 10^5$
--

$f_e$ 250 = 0.13d $f_e$ 415 = 0.48d $f_e$ 500 = 0.46d
---

→ Percentage of steel (%):—

$$\Rightarrow T = C$$

$$\Rightarrow 0.87 f_y A_{st} = 0.36 f_{ck} A_{cu} b$$

$$\Rightarrow A_{st} = \frac{0.36 f_{ck} A_{cu} b}{0.87 f_y}$$

$P_t = \frac{A_{st}}{b d}$
----------------------------

$$\Rightarrow \frac{A_{st}}{b} = \frac{0.36 f_{ck} A_{cu}}{0.87 f_y}$$

$$\Rightarrow \frac{A_{st}}{b d} \times d = \frac{0.36 f_{ck} A_{cu}}{0.87 f_y}$$

$\Rightarrow \frac{A_{st}}{b d} = \frac{0.36 f_{ck} A_{cu}}{0.87 f_y d}$
--

→ Limiting percentage of steel :-

$P_{lim}(\%)$

$$= \left( \frac{0.36 f_{ck}}{0.87 f_y} \cdot \frac{a_{umax}}{d} \times 100 \right) \%$$

Balanced, under & over reinforced sections

$\left\{ \begin{array}{l} a_u = a_{umax} \text{ (Balanced)} \rightarrow M_{ulim} \\ a_u < a_{umax} \text{ (Under)} \rightarrow M_{ui} \\ a_u > a_{umax} \text{ (Over)} \rightarrow M_{ulim} \end{array} \right.$

$$M_{ui} = T \times lever \text{ arm}$$

$$M_{ulim} = C \times lever \text{ arm}$$

Q:- Determine the depth of neutral axis of a beam  $250\text{mm} \times 350\text{mm}$  reinforced with 4 bars of  $16\text{mm}$  diameter also check for the type of section & find out the moment of resistance. Use M20 concrete & Fe 500 steel.

Sol:-

$$A_{st} = \text{no. of bars} \times \frac{\pi}{4} \times (d)^2$$

$$= 4 \times \frac{\pi}{4} \times (16)^2$$

$$= 804.24$$

$$\begin{aligned}
 \mu_u &= \frac{0.87 f_y A_{st}}{0.36 f_{ck} b} \\
 &= \frac{0.87 \times 500 \times 804.24}{0.36 \times 10 \times 250} \\
 &= 388.716
 \end{aligned}$$

$$\begin{aligned}
 \mu_{u, \max} &= 0.46 d \\
 &= 0.46 \times 350 \\
 &= 161
 \end{aligned}$$

$\mu_u > \mu_{u, \max}$  (Over reinforced).  
 The section should be ~~over~~ redesigned.  
 Therefore,  $\mu_{lim}$  should be calculated.

$$\begin{aligned}
 \mu_{lim} &= c \times \text{lever arm} \\
 &= 0.36 f_{ck} \cdot \mu_{u, \max} \cdot b \times (d - 0.42 \mu_{u, \max}) \\
 &= 0.36 \times 10 \times 161 \times 250 \times (350 - 0.42 \times 161) \\
 &= 144900 \times 282.28 \\
 &= 40916862 \text{ Nmm} \\
 &= 40.916862 \text{ kNm}
 \end{aligned}$$

Q:- Determine the M.O.R of a beam having dimension as 250 mm x 360 mm the area of steel consists of 3 bars of 20 mm diameter placed at a distance of 40 mm from bottom of the beam. Use M30 concrete & Fe 415 steel.

$$\begin{aligned}
 \text{Ans:-} \quad b &= 250 \text{ mm} \\
 d &= 360 - 40 \\
 &= 320 \text{ mm}
 \end{aligned}$$

$$-Asl = \text{no. of bars} \times \frac{\pi}{4} \times (d)^2$$

$$= 3 \times \frac{\pi}{4} \times (20)^2$$

$$= 942.47$$

$$\mu_u = \frac{0.87 f_y A_s}{0.36 f_{ck} b}$$

$$= \frac{0.87 \times 415 \times 942.47}{0.36 \times 20 \times 250}$$

$$= 126.03$$

$\mu_{lim} = 0.48$  (under reinforced)  
 $\mu_u < \mu_{lim}$  (under reinforced)

$$M_u = 0.87 f_y A_s x [d - 0.42 \mu_u]$$

$$= 0.87 \times 415 \times 942.47 \times [250 - 0.42 \times 126.03]$$

$$= 340278.79 \times 267.11$$

$$= 90878801.83 \text{ Nmm}$$

$M_u = 90878801.83 \text{ Nmm}$   
 $M_u = 90.8788 \text{ kNm}$

Q:- A rectangular beam is 20cm wide & 40cm deep of to the centre of reinforcement. find the area of reinforcement required if it has a revised of moment of 25 kNm. Use M20 concrete and Fe415 steel.

Sol:-  
 $b = 20\text{cm} = 200\text{mm}$   
 $d = 40\text{cm} = 400\text{mm}$

$M_u = 1.5 \times 25 \times 10^6$   
 $= 37500000$

$\Rightarrow M_u = T \times L \times A$   
 $= 0.87 f_y A_{st} (d - 0.42 x_u)$   
 $= 0.87 \times 415 \times A_{st} (400 - 0.42 x \frac{0.87 f_y A_{st}}{1440})$   
 $= 361.05 A_{st} (400 - 0.42 x \frac{361.05 A_{st}}{1440})$

$= 361.05 A_{st} \times 400 - 361.05 A_{st} (0.42 x \frac{361.05 A_{st}}{1440})$   
 $\Rightarrow 37500000 = 361.05 A_{st} \times 400 - 361.05 A_{st} (0.42 x \frac{361.05 A_{st}}{1440})$

$\Rightarrow 37500000 = 361.05 A_{st} \times 400 - 361.05 A_{st} \times 0.125 A_{st}$

$\Rightarrow 37500000 = 361.05 A_{st} \times 400 - 39.72 A_{st}^2$

$\Rightarrow 37500000 + 39.72 A_{st}^2 = 144420 A_{st}$

$\Rightarrow 39.72 A_{st}^2 - 144420 A_{st} + 37500000 = 0$

$\Rightarrow A_{st} = 281.444$

$\Rightarrow A_{st} = 281$



$$\mu = \frac{0.87 f_y A_{st}}{0.36 f_{ck} b}$$

$$= \frac{0.87 \times 415 \times 281}{0.36 \times 20 \times 200}$$

$$= 70.45$$

Q:- A singly reinforced beam section 250 x 550 mm is reinforced in tension side with 1256 mm<sup>2</sup> of 415 steel with effective cover of 50 mm. Determine the uniformly distributed load it can carry if span of the beam is 6.5 m. Use M25 concrete.

Sol:-  $b = 250 \text{ mm}$   
 $d = 550 - 50 = 500 \text{ mm}$

$$\mu = \frac{0.87 f_y A_{st}}{0.36 f_{ck} b}$$

$$= \frac{0.87 \times 415 \times 1256}{0.36 \times 25 \times 250}$$

$$= 201.57$$

$$\mu_{max} = 0.48d$$

$$= 0.48 \times 500$$

$$= 240 \text{ mm}^2$$

$\mu < \mu_{max}$  (under reinforced)

$$\begin{aligned} \Rightarrow M_u &= T \times \text{lever arm} \\ &= 0.87 f_y A_{st} a (d - 0.42 a) \\ &= 0.87 \times 415 \times 1256 (200 - 0.42 \times 201.54) \\ &= 453478.8 \times 415 \cdot 35 \\ &= 188352419.6 \end{aligned}$$

$$\boxed{UDL = \frac{wL^2}{8}}$$

$$\Rightarrow \frac{wL^2}{8} = 114$$

$$\Rightarrow \frac{w \times (5.5)^2}{8} = 188352419.6$$

~~$$\Rightarrow \frac{w \times 5.5^2}{8} = 188352419.6$$~~

$$\Rightarrow w \times (5.5)^2 = 188352419.6 \times 8$$

$$\Rightarrow w = \frac{188352419.6 \times 8}{(5.5)^2}$$

$$= \frac{1506819357}{30.25}$$

$$= 49812210.14$$

$$= 49.81 \text{ kNm}$$

Dt-14/02/20

Q: Using ISM, find the moment of resistance of a singly reinforced beam 200mm wide & 417mm deep of to the centre of the tensile reinforcement. Though overall depth of the beam is 450mm. The beam is reinforced with 3 bars of 16mm diameter. The concrete used is M20 & steel used

in detail:  
sol:  $b = 300 \text{ mm}$   
 $d = 417 \text{ mm}$

$$\rightarrow A_{st} = 3 \times \frac{\pi}{4} \times (16)^2 = 603.18$$

$$\rightarrow m_u = \frac{0.87 f_y A_{st}}{0.36 f_{ck} b} = \frac{0.87 \times 415 \times 603.18}{0.36 \times 25 \times 300} = 100.82$$

$$\Rightarrow m_{u, \max} = 0.48 d = 0.48 \times 417 = 200.16$$

$m_u < m_{u, \max}$  (under reinforced)

$$\rightarrow M_u = T \times \text{lever arm} = 0.87 f_y A_{st} (d - 0.42 m_u) = 0.87 \times 415 \times 603.18 (417 - 0.42 \times 100.82) = 217778.14 \times 374.65 = 81590560.15 = 81.59 \text{ kNm}$$

Date 14/02/20  
Page 26

Q:- A singly reinforced beam of cross-section 300mm x 600mm is simply supported over the clear span of 4.25m with support of 300mm each. Calculate ultimate moment of resistance. Use M20 concrete & Fe 415 steel.

Sol:-  $b = 300 \text{ mm}$   
 $d = 600 - 50 = 550 \text{ mm}$   $\therefore$  Assume 50mm eff. cover.

$$\Rightarrow \alpha_{max} = 0.48d$$
$$= 0.48 \times 550$$
$$= 264$$

$$\Rightarrow M_{ulim} = C \times lever \text{ arm}$$
$$= 0.36 \times f_y \times A_s \times (d - \alpha_{max})$$
$$= 0.36 \times 20 \times 200 \times (550 - 0.48 \times 550)$$
$$= 0.36 \times 20 \times 264 \times 300 \quad (550 - 0.48 \times 264)$$
$$= 250.403788.8$$
$$\approx 250.4 \text{ kNm}$$

$$\text{Span length (eff.)} = 4.25 + 0.3$$
$$= 4.55 \text{ m}$$

Q:- Find the moment of resistance of a RCC beam 230mm wide & 450mm deep. The beam is reinforced with 4-16mm diam bars in tension zone. The eff. cover to the reinforcement is 40mm. Grade of concrete & steel are M20 & Fe 415, respectively.

Sol:  $b = 23 \text{ cm} = 230 \text{ mm}$   
 $d = 45 \text{ cm} = 450 \text{ mm} - 40 \text{ mm}$   
 $= 410 \text{ mm}$

$$\Rightarrow A_{st} = 4 \times \frac{1}{4} \times (16)^2$$

$$= 804.25$$

$$\Rightarrow m_u = \frac{0.87 f_y A_{st}}{0.36 f_c b}$$

$$= \frac{0.87 \times 415 \times 804.25}{0.36 \times 20 \times 230}$$

$$= 175.25$$

$$\Rightarrow a_{umax} = 0.48 d$$

$$= 0.48 \times 410 = 196.8 \text{ mm}$$

$a_{umax} < a_{umax} \text{ (under reinforced)}$

$$M_u = T \times \text{lever arm}$$

$$= 0.87 f_y A_{st} \times d = 0.42 m_u$$

$$= 0.87 \times 415 \times 804.25 \times (410 - 0.42 \times 175.25)$$

$$= 229766432 \text{ Nmm}$$

$$= 229.767 \text{ kNm}$$

## Chapter - 04

Date: \_\_\_\_\_  
Page: \_\_\_\_\_

→ Shear stress in R.C.C. beam :-

- ) RCC is a composite material that's way the exact shear distribution as per elastic theory is very complex (difficult).
- ) It is parabolic in the compression zone with 0 at the top & max at the neutral axis. The value of shear stress is const. in the tensile zone & is equal to the max shear stress (q) because the concrete below the neutral axis (tensile zone) is assume to be cracked & neglected so, the max value of shear stress (q) as per elastic theory is given by,

$$q = \frac{V}{bjd}$$

where,

V = shear force at the section

b = breadth of the section

d = depth

j = lever arm depth factor



shear stress in R.C.C. beam

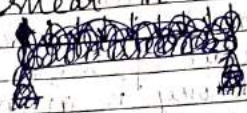
→ Nominal shear stress ( $\tau_v$ ):

$$\tau_v = \frac{V}{bd}$$

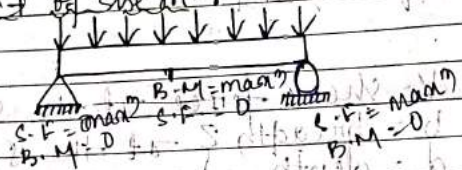
where, V: shear force

→ Diagonal Tension:

It is a type of tensile which caused in the tensile zone of the beam due to shear at or near the supports.



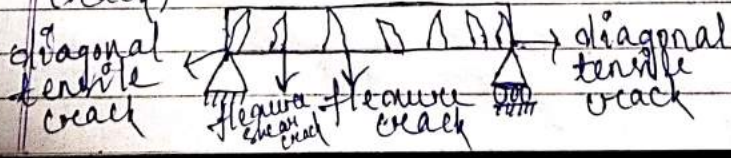
→ Effect of shear:



Notes:-

Load which acts vertically on a surface of a beam is called as transverse load.

- At or near the mid span the cracks will be vertical named as :- flexure cracks due to bending along.
- At or near the supports the cracks are inclined at  $45^\circ$ . (shear or diagonal tensile crack).



In bet<sup>n</sup> - the supports, & mid span - the crack inclination varies from 45° to 90°.

Types of shear reinforcement :-  
There are 3 types of shear reinforcement used :-

- (i) Vertical stirrups
- (ii) Bent up bars along with stirrups.
- (iii) Inclined stirrups.

Dt - 02/03/20

(i) Vertical stirrups :-

These are the steel bars vertically placed around the tensile reinforcement at suitable spacing along the length of the beam.

Their diameter varies from 6mm to 16mm

The free ends of the stirrups are anchored in the compression zone of the beam to the anchored bars (chairs bars) or the compressive reinforcement

Vertical stirrups are provided to resist the shear force & these are provided depending upon the magnitude of shear force. These are of one legged.

The figures of vertical stirrups are as listed below :-



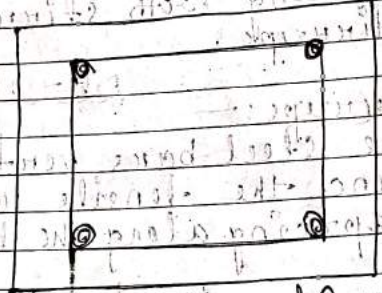
(b) (c) (d)

10 .pst



Fig. 01

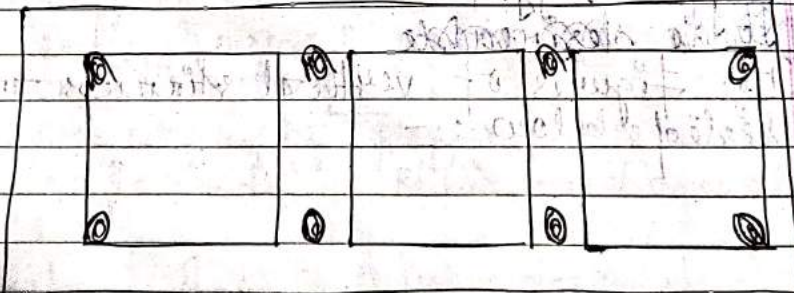
(One legged)



(Two legged)



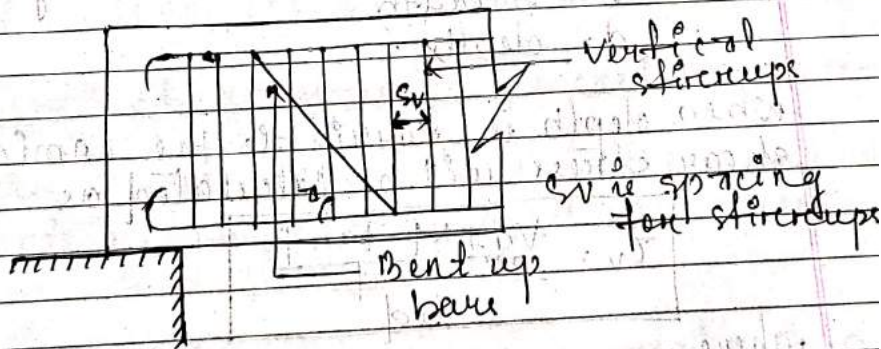
(Four legged)



(Six legged)

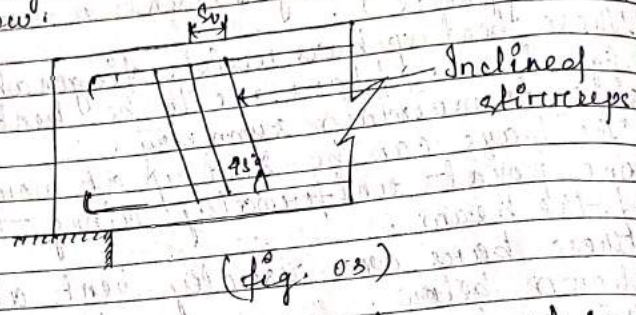
Fig. 01

- Date \_\_\_\_\_  
Page \_\_\_\_\_
- Bent up bars along with vertical stirrups:
  - || Some of the longitudinal bars in a beam can be bent up near the supports where they are not required to resist bending moment. (Bending moment is very less near the supports).
  - || These bent up bars resist diagonal tension.
  - || Equal no. of bars are to be bent on both sides to maintain symmetry.
  - || The bars can be bent up at more than one point uniformly along the length of the beam.
  - || These bars are usually bent at  $45^\circ$  as shown below.
  - || This system is used for heavier shear force.
  - || The total shear resistance of the beam is calculated by adding the contribution of bent up bars & vertical stirrups, where contribution of bent up bars should not be greater than half of the shear reinforcement.



(fig. 02)

→ Inclined stirrups:-  
 These are provided generally at 45° angle for resisting diagonal tension & provided throughout the length of the beam as given below:



→ Design of shear reinforcement:-

(a) Nominal shear stress:-  
 The nominal shear stress in beams and slabs of uniform depth is calculated as

$$\tau_v = \frac{V_u}{bd}$$

where,  
 $V_u$  = shear force due to design loads.  
 $b$  = breadth.  
 $d$  = depth.

When depth is variable the nominal shear stress will be calculated as

$$\tau_v = \frac{V_u \pm M \cdot \tan \beta}{b'd}$$

where,  
 $M$  = bending moment.

$\theta$  = Angle bet<sup>n</sup> top & bottom edges of the beam.

Notes:-

The (-) sign in the formulae is applied when bending moment (M) increases numerically in the same direction as the e.f. depth increases & the (+) sign when the bending moment (M) decreases numerically in the direction...

→ Percentage of steel (p<sub>t</sub>): -

$$p_t = \frac{100 \cdot A_s}{bd}$$

In case of solid slab (k)  
The shear strength of solid slab =  $k \cdot \tau_c$   
where the value of k is given below.

Overall depth of slab in mm	300 or more	275	250	225	200	175	150 or less
Value of k	1.100	1.05	1.10	1.15	1.20	1.25	1.30

→ max<sup>m</sup> shear stress in concrete ( $\tau_{max}$ ): -

1<sup>st</sup> → The nominal shear stress in the beam with shear reinforcement shall not exceed max<sup>m</sup> shear stress.

2<sup>nd</sup> → If nominal shear stress is greater than the max<sup>m</sup> shear stress then the section is

To be redesigned.

3)  $\rightarrow$  If the shear strength of the concrete beam is less than nominal shear stress due to the loads coming on the beam then shear reinforcement is to be provided.

Grade of concrete	M15	M20	M25	M30	M35	M40
$\tau_c$ (N/mm <sup>2</sup> )	2.5	2.8	3.1	3.5	3.7	4.0

- min<sup>m</sup> shear reinforcement :-
- When the nominal shear stress ( $\tau_v$ ) is less than the design shear strength ( $\tau_c$ ) or shear strength of concrete, then no shear reinforcement is to be design.
  - In such cases, min<sup>m</sup> shear reinforcement is to be provided in the form of stirrups such as

$$\frac{A_{sv}}{b \cdot s_v} \geq \frac{0.4}{0.87 f_y}$$

where

$A_{sv}$  = total cross-sectional area of stirrup legs eff. in shear.

$s_v$  = spacing of stirrups along the length of the member.

$b$  = breadth of the beam.

In case of flanged beam breadth of web.

$f_y$  = characteristic strength of compressive strength of the stirrup reinforcement which

shall not be greater than  $45 \text{ N/mm}^2$ .

→ Max<sup>m</sup> spacing of stirrups:—

✓ The max<sup>m</sup> spacing of vertical stirrups shall not exceed  $0.75 d$  or  $300 \text{ mm}$  whichever is less.

✓ In case of inclined stirrups at  $45^\circ$  the max<sup>m</sup> spacing should not be greater than  $d$  or  $300 \text{ mm}$  whichever is less.

→ Design of shear reinforcement ( $V_{sw}$ ):—

✓ When  $(\tau_v)$  exceed  $(\tau_c)$  shear reinforcement is to be designed & can be provided in the following forms:—

(i) Vertical stirrups.

(ii) Bent up bars along with stirrups.

✓ Shear reinforcement is provided to carry a shear force ( $V_{sw}$ ).

$$V_{sw} = V_u - \tau_c \cdot b \cdot d$$

where,

$V_u$  = Shear force after design loads.

$\tau_c \cdot b \cdot d$  = Shear resistance of the concrete section.

(i) Vertical stirrups:—

The spacing of the vertical stirrups is given  $V_{sw} = \frac{0.87 f_y A_{sv}}{s_v} \cdot d$

where,

$A_{sv}$  = total cross-sectional area

$s_w$  = spacing of stirrups.  
 $f_y$  = characteristic strength of  
 stirrups.  
 $d$  = eff. depth.

- (ii) Bent up bars along with stirrups ( $V_{cu}$ ):—
- (i) When bent up bars are provided, their contribution towards shear resistance should not be more than half of the total shear capacity of the member. It means that their contribution should not be more than 50% of the total shear capacity.
  - (ii) Shear force taken by bent up bars (at the same connection) is calculated as follows:

$$V_{cu} = 0.87 f_y A_{sv} \sin \alpha$$

where,

$$V_{cu} \neq V_{cu}/2$$

$A_{sv}$  = Area of bent up bars.

$\alpha$  = angle bet<sup>n</sup> bent up bars & member axis.

- (iii) Combined systems:—
- In this system first the contribution of bent up bars ( $V_{cu}$ ) is calculated then vertical stirrups are designed for remaining shear force.

$$V_{cu} - V_{cu}$$

→ Steps for design of shear reinforcement :-  
Given data,

- Load
- span of beam
- Material (grade of concrete & steel)
- Area of tensile steel ( $A_{st}$ ).

Procedure :-

- (1) Calculate shear force ( $V_u$ ) at critical section of the beam due to given loads.
- (2) Determine nominal shear stress ( $\tau_v$ )

$$\tau_v = \frac{V_u}{bd}$$

- (3) Depending on the grade of concrete & % of steel

$$\frac{100 A_{st}}{bd}$$

find out shear strength of concrete ( $\tau_c$ ) in table no. 19 of IS 456.

- (4) Compare  $\tau_v$  &  $\tau_c$  (max shear strength) if  $\tau_v > \tau_c$  then re-design the section.

- (5) Compare  $\tau_v$  &  $\tau_c$
- (6) If  $\tau_v < \tau_c$  then nominal shear reinforcement is to be provided in the form of vertical stirrups. The spacing of vertical stirrups is given as,

$$\frac{A_{sv}}{b \cdot s_v} \geq \frac{0.4}{0.87 f_y}$$



(ii) If  $\tau_v > \tau_c$  then shear reinforcement is to be design as follows:-  
 → Calculate

$$V_{us} = V_u - \tau_c b d$$

→ If vertical stirrups are provided then their spacing is governed by the following eq<sup>n</sup>.

$$V_{us} = \frac{0.87 f_y A_{sv} d}{s_v}$$

→ If bend up bars are also used then 1st & 2nd all complete the shear force taken by bend up bars  $V'_{us}$  &  $V_{us} \times V_{us}/2$ .

$$V'_{us} = 0.87 f_y A_{sv} \times \sin \alpha$$

→ For the balance shear reinforcement ( $V_{us} - V'_{us}$ ), design the vertical stirrups.

$$V_{us} - V'_{us}$$

(6) The spacing of stirrups should not exceed 0.75 of or 300mm which ever is less.

(7) The spacing of stirrups can be varied along the length of the beam by calculating the distance from the support upto which shear reinforcement is to be design & in the rest of the length min<sup>m</sup> shear reinforcement may provided.

Q1:- An RCC beam 300mm x 600mm in section is reinforced with 5-25 mm  $\phi$  bars (L.F.). It is subjected to redesign shear force of 200 kN. Comment on its shear design. Use M20 concrete & Fe 415 steel.

Sol: Given data,

$$b = 300 \text{ mm}$$

$$d = 600 \text{ mm}$$

$$f_y = 415$$

$$f_{ck} = 20$$

$$V_u = 200 \text{ kN} = 200 \times 10^3 \text{ N}$$

$$\Rightarrow A_{st} = \frac{V_u}{f_y} \times \frac{1}{4} \times (2d)^2$$

$$= \frac{200 \times 10^3}{415} \times \frac{1}{4} \times (1200)^2$$

$$\Rightarrow \tau_v = \frac{V_u}{b d}$$

$$= \frac{200 \times 10^3}{300 \times 600}$$

$$= 1.11$$

$$\Rightarrow \% \text{ of steel (pt)} = \frac{100 A_{st}}{b d}$$

$$= \frac{100 \times 2454.37}{300 \times 600}$$

$$= 1.36\%$$

→ for M20 grade concrete, design shear strength of concrete,  $\tau_c = 0.72$

→ for M20 grade concrete, max<sup>m</sup> shear strength,  $\tau_{cmax} = 2.8$

→ After comparing the values of  $\tau_v$  &  $\tau_{cmax}$  we get,

$$\tau_{cmax} > \tau_v$$

Hence, O.K.

After comparing the values of  $\tau_v$  &  $\tau_c$  we got,  $\tau_v > \tau_c$

Hence, shear design is required.

Q2: An RCC beam  $200\text{mm} \times 400\text{mm}$  of  $f_c$  carrying a uniformly distributed load of  $70\text{ kN/m}$  over a clear span of  $6\text{m}$ . The beam is reinforced with  $1\%$  steel on tension side. Comment on the shear design of the beam. Use M20 concrete & load factor  $1.5$ .

Sol: Given,

$$b = 200\text{mm}$$

$$d = 400\text{mm}$$

$$w = 70\text{ kN/m}$$

$$l = 6\text{m}$$

$$\text{load factor } (\tau_f) = 1.5$$

$$\begin{aligned} \Rightarrow \text{factored load } (w_u) &= w \times \tau_f \\ &= 70 \times 1.5 \\ &= 105\text{ kN/m} \\ &= 105 \times 10^3\text{ N/m} \end{aligned}$$

$$\begin{aligned} \Rightarrow V_u &= \frac{w_u l}{2} \\ &= \frac{105 \times 10^3 \times 6}{2} \\ &= 315000\text{ N} \end{aligned}$$

$$\begin{aligned} \Rightarrow \tau_v &= \frac{V_u}{bd} \\ &= \frac{315000}{200 \times 400} \\ &= 3.94 \end{aligned}$$

→ For M20 grade concrete <sup>min</sup> design shear strength,  $\tau_{cmax} = 2.8$

After comparing the values of  $\tau_v$  &  $\tau_{cmax}$  we got,

$$\tau_v > \tau_{cmax}$$

Hence, this section should be re-designed i.e. its dimension should be ~~revised~~ revised.

→ For M20 grade concrete <sup>design</sup> shear strength of concrete  $\tau_c = 0.62$

Q3:- A simply supported RCC beam to be 250mm wide & 450mm deep (eff.) is reinforced with 4-18mm diameter bars. Design the shear reinforcement if M20 grade of concrete & Fe 415 steel is used & beam is subjected to a shear force of 150kN at service state.

Sol:- Given,

$$b = 250 \text{ mm}$$

$$d = 450 \text{ mm}$$

$$f_{ck} = 20$$

$$f_y = 415$$

$$V = 150 \text{ kN}$$

$$\Rightarrow V_{cr} = 150 \times 1.5$$
$$= 225 \text{ kN}$$
$$= 225 \times 10^3 \text{ N}$$

$$\Rightarrow A_{st} = 4 \times \frac{\pi}{4} \times (18)^2$$
$$= 1017.87$$

$$\Rightarrow \tau_v = \frac{V_{cr}}{b d}$$
$$= \frac{225 \times 10^3}{350 \times 450}$$
$$= 2$$

$$\Rightarrow \% \text{ of steel (pt)} = \frac{100 A_{st}}{b d}$$
$$= \frac{100 \times 1017.87}{250 \times 450}$$

$\Rightarrow$  For M20 grade concrete, min. shear strength

$\rightarrow$  After computing the value of  $\tau_v$ ,  $\tau_{max}$  we got,

$$\tau_{max} > \tau_v$$

Hence, O.K.

→ For M20 grade concrete, design shear strength of concrete,  
 $\tau_c = 0.62$

After comparing the value of  $\tau_v$  &  $\tau_c$  we get,

$$\tau_v > \tau_c$$

→ Design of shear reinforcement,

$$\begin{aligned} \Rightarrow V_{cu} &= V_{ci} - \tau_c b d \\ &= 225 \times 10^3 - 0.62 \times 250 \times 450 \\ &= 155250 \end{aligned}$$

Assuming 8mm - 2 legged stirrups.

$$\begin{aligned} \Rightarrow A_{sv} &= 2 \times \frac{\pi}{4} \times (8)^2 \\ &= 100.53 \end{aligned}$$

$$\Rightarrow V_{cu} = \frac{0.87 f_y A_{sv} d}{s_v}$$

$$\Rightarrow s_v = \frac{0.87 f_y A_{sv} d}{V_{cu}}$$

$$\Rightarrow s_v = \frac{0.87 \times 115 \times 100.53 \times 450}{155250}$$

$$s_v = 1.0520$$

$$s_v = 1103/20$$

$$= \frac{A_{sv}}{b \cdot s_v} \geq \frac{0.4}{0.87 f_y}$$

$$\Rightarrow \frac{A_{sv}}{b \cdot s_v} = \frac{0.4}{0.87 f_y}$$

$$\Rightarrow A_{sv} \cdot 0.87 f_y = 0.4 b \cdot s_v$$

$$\Rightarrow s_v = \frac{A_{sv} \cdot 0.87 f_y}{0.4 b}$$

$$= \frac{100.53 \times 0.87 \times 415}{0.4 \times 250}$$

$$= 362.96$$

check for spacing.

The spacing of stirrups should be min<sup>m</sup> from the following.

(i)  $s_v = 0.75 d$   
 $= 0.75 \times 450$   
 $= 337.5$

(ii)  $s_v = 300 \text{ mm}$

(iii)  $s_v = 105.20$

(iv)  $s_v = 362.96$

∴ Provide 8mm - 2 legged stirrups through out the length of the beam @ 100mm c/c

Q:- A simply supported beam 300mm x 600mm eff. is reinforced with 5 bars of 25mm dia. It carries a uniformly distributed load of 80 kN/m (including its own weight). Over an eff. span of 6m. Out of 5 main bars 2 bars can be bent up safely near the supports. Design the shear reinforcement for the beam. Use M20 grade of concrete & Fe 415 steel.

sol:- Given,

$b = 300\text{mm}$

$d = 600\text{mm}$

$w = 80\text{ kN/m}$

$l(\text{eff.}) = 6\text{m}$  (∵ assumed 400mm as clear cover)

$l = 6000 - 400$

$= 5600\text{ mm}$

$= 5.6\text{ m}$

$f_{ck} = 20$

$f_y = 415$

assumed load factor = 1.5

$\Rightarrow f_{st} = 3 \times \frac{\pi}{4} \times (25)^2$   
 $= 1472.62$

$\Rightarrow$  factored load ( $w_u$ ) =  $w \times 1.5$   
 $= 80 \times 1.5$   
 $= 120\text{ kN/m}$

$= 120 \times 10^3\text{ N/m}$



⇒ factored shear force  
( $V_u$ ) =  $\frac{w_u l}{2}$

$$= \frac{120 \times 10^3 \times 5.6}{2}$$

$$= 336000 \text{ N}$$

$$\Rightarrow \tau_v = \frac{V_u}{b d}$$

$$= \frac{336000}{300 \times 600}$$

$$= 1.86 \text{ N/mm}^2$$

$$\Rightarrow \% \text{ of steel (pt)} = \frac{100 A_{st}}{b d}$$

$$= \frac{100 \times 1472.62}{300 \times 600}$$

$$= 0.81 \%$$

⇒ for M20 grade concrete, max<sup>n</sup> shear strength,  
 $\tau_{cmax} = 2.8 \text{ N/mm}^2$

After comparing the values of  $\tau_v$  &  $\tau_{cmax}$  we get,

$$\tau_v < \tau_{cmax}$$

Hence, this section should be re-designed i.e. its dimension should be reversed.

⇒ for M20 grade concrete design shear strength of concrete,

$$\tau_c = 0.62$$

- After comparing the values of  $\tau_v$  &  $\tau_c$  we get,  
 $\tau_v > \tau_c$

⇒ design of shear reinforcement.

$$\Rightarrow V_{cs} = V_u - \tau_c \cdot b \cdot l$$
$$= 33600 - 0.62 \times 300 \times 600$$
$$= 22440$$

$$= \frac{V_{cs}}{2} = \frac{22440}{2}$$
$$= 11220$$

~~assuming 8mm dia vertical stirrups~~

~~at 100mm spacing~~

Dt = 13/03/20

⇒ for bent up bars ( $V'_{cs}$ )

$$= 0.87 f_y A_w \times \sin \alpha$$
$$= 0.87 \times 415 \times \frac{1}{4} \times (25)^2 \times \sin 45^\circ$$
$$= 125320.54$$

for 2 bent up bars

$$= 2 \times 125320.54$$
$$= 250641.08$$

But  $V'_{cs} \neq V_{cs}/2$   
 $250641.08 \neq 11220$

Balance shear force is carried out for vertical stirrups.

$$= V_{cs} - V'_{cs}$$
$$= 22440 - 11220$$

$$= 112200$$

Assume 2-legged stirrup of 8mm dia

$$\Rightarrow A_{sv} = 2 \times \frac{\pi}{4} \times (8)^2$$

$$= 100.53$$

$$\Rightarrow V_{uc} = \frac{0.87 f_y A_{sv} d}{s_v}$$

$$\Rightarrow s_v = \frac{0.87 f_y A_{sv} d}{V_{uc}}$$

$$\Rightarrow s_v = \frac{0.87 \times 415 \times 100.53 \times 600}{224400}$$

min<sup>m</sup> shear reinforcement

$$\frac{A_{sv}}{b \cdot s_v} \geq \frac{0.4}{0.87 f_y}$$

$$\Rightarrow \frac{A_{sv}}{b \cdot s_v} = \frac{0.4}{0.87 f_y}$$

$$\Rightarrow A_{sv} \cdot 0.87 f_y = 0.4 b \cdot s_v$$

$$\Rightarrow s_v = \frac{A_{sv} \cdot 0.87 f_y}{0.4 b}$$

$$= \frac{100.53 \times 0.87 \times 415}{0.4 \times 300}$$

$$= 302.47$$

Check for spacing :-  
The spacing of stirrups should be min<sup>m</sup> from the following.

(i)  $s_v = 0.75 d$   
 $= 0.75 \times 600$   
 $= 450$

(ii)  $s_v = 300 \text{ mm}$

(iii)  $s_v = 302.47$

(iv)  $s_v = 129.67$

∴ Provide 8mm - 2 legged stirrups throughout the length of the beam @ 100mm/c.

→ Bond :-

- ii In case of concrete, the dem bond resists to the adhesion bet<sup>n</sup> concrete & steel which resists the slip of steel bar from the concrete.
- iii It is the bond which is responsible for transfer of stresses from steel to concrete.
- iv It is a composite action on steel & concrete.
- v The bond develops due to setting of concrete on drying which results in gripping of the steel bars.

→ The bond resistance in reinforced concrete is obtained by the following mechanism :-

1st :- Chemical adhesion

It is due to gum like property of the substance formed after setting of concrete.

2nd:- frictional Resistance:-

It is due to friction bet<sup>n</sup> steel & concrete.

3rd:- Gripping action:-

It is due to gripping of steel by concrete on drying.

4th:- Mechanical interlock:-

It is provided by the corrugation or ribs present on the surface of all forms of bars.

→ Bond bet<sup>n</sup> steel & concrete can be increased by the following methods:-

- (1) Using deformed or twisted bars.
- (2) Using rich mix of concrete.
- (3) Adequate compaction & curing of concrete for proper setting.
- (4) Providing hooks at the end of the reinforcement bars.

DT-08/04/20

→ Bond stress:-

The stress which acts on the outer interface of steel to the surrounding of concrete is called as bond stress.

- It helps in keeping bond bet<sup>n</sup> reinforcement & concrete together.

- Bond stress resists any force that tries to pull out the steel or reinforced bar.

from the concrete.  
- It depends upon grade of concrete & type of reinforcement.

→ Types of bond :-  
Basically it is of 2 types :-

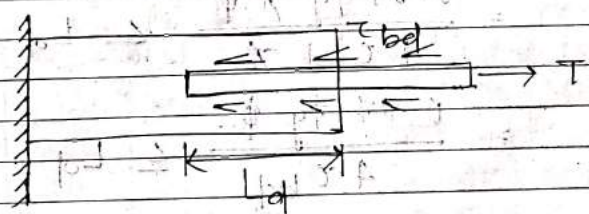
- (i) Anchorage bond
- (ii) Flexural bond.

(i) Anchorage bond :-

- It is provided in case of axial tension or axial compression.  
- When a bar carrying certain force is removed in such cases it is necessary to transfer the force from the bars to the surrounding of concrete over a certain length. It is also called as anchorage length.

(ii) Development length in tension :-

- In case of flexural tension or flexural compression the length of bar required to transfer the force from the bar to the surrounding of concrete over a certain length is called development length.  
- It is determined by performing 'cool out test'.



where,

$L_d$  = development length.

$\tau_{bd}$  = bond stress.

$T$  = tensile force.

Let, us consider a steel bar embedded in concrete as shown in above fig.

The bar is subjected to a tensile force  $T$ . Due to this tensile force, the steel bar will tend to come out or slip out of the concrete. This tendency of slipping is resisted by the bond stress developed over the surface of the bar. To avoid slipping,

$$T \leq \tau_{bd} \times 2 \times \frac{\phi}{2} \times L_d$$

Surface area =  $2 \pi \times L_d$

Tensile force ( $T$ ) =  $0.87 f_y \times \frac{\pi}{4} \times \phi^2$

$$0.87 f_y \times \frac{\pi}{4} \times \phi^2 \leq \tau_{bd} \times 2 \times \frac{\phi}{2} \times L_d$$

$$0.87 f_y \times \frac{\pi}{4} \times \phi^2$$

$$\leq \tau_{bd} \times \pi \times \phi \times L_d$$

$$0.87 f_y \times \frac{\pi}{4} \times \phi^2 \times \frac{4}{\pi \times \phi} \leq L_d$$

$$= 0.87 f_y \times \phi$$

$$\leq L_d$$

$$= \frac{0.87 f_y \phi}{A \tau_{bd}}$$

$$\leq L_d$$

$$= L_d \geq \frac{0.87 f_y \phi}{4 \tau_{bd}}$$

$$\Rightarrow L_d = \frac{0.87 f_y \phi}{4 \tau_{bd}} \quad (\text{tension})$$

→ Permissible bond stress for plain bars & deformed bars in tension :-

Grade of concrete	$\tau_{bd}$ for plain bars (N/mm <sup>2</sup> )	$\tau_{bd}$ for deformed bars (N/mm <sup>2</sup> )
M20	1.2	1.92
M25	1.4	2.24
M30	1.6	2.4
M35	1.7	2.72
M40 & above	1.9	3.04

→ Development length in compression :-

$$= \frac{0.87 f_y \phi}{4 (1.25 \tau_{bd})}$$

$$L_d = \frac{0.87 f_y \phi}{5 \tau_{bd}} \quad (\text{compression})$$

Type of steel	$f_y$ N/mm <sup>2</sup>	L <sub>d</sub> for single bar :-					
		L <sub>d</sub> in tension			L <sub>d</sub> in compression		
Fe 250	250	M25	M20	M25	M15	M20	M25
Fe 415	415	55φ	46φ	39φ	44φ	37φ	31φ
Fe 500	500	56φ	47φ	40φ	45φ	38φ	32φ
		69φ	58φ	49φ	54φ	46φ	39φ



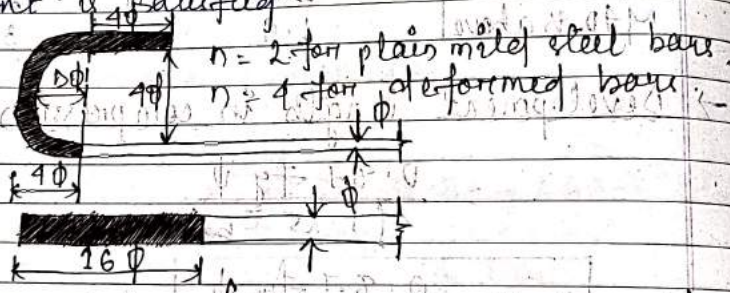
N.B.  $\phi$  = diameter of the bar.

- Anchorage for reinforcement bars :-
- The development length of bars can be provided in the form of straight bars if available otherwise it may be provided in the form of straight & partially hooked.
  - The anchorage is normally provided in the form of bends & hooks.

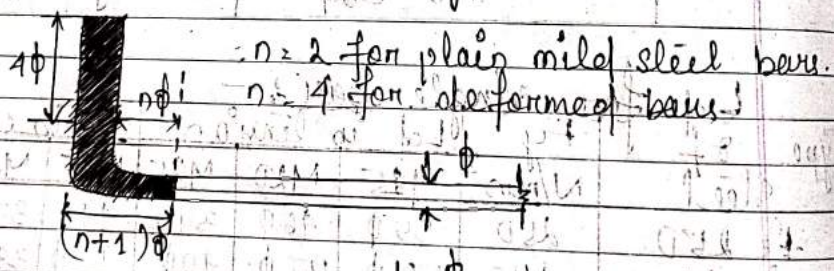
→ Anchorage bars in tension :-

(i) Generally hooks are provided in plain bars in tension.

(ii) Deformed bars may be used without end anchorage if the development length requirement is satisfied.



(a) Anchorage value =  $16\phi$



(b) Anchorage value =  $8\phi$

(iii) The hook should always satisfied the specification.

(iv) The anchorage value of a bend shall be taken as 4 times of the diameter of the bar for each  $45^\circ$  bend subject to a max<sup>m</sup> of 16 times of the diameter of the bar.

(v) The anchorage value of a standard 12-type hook is equal 16 times of the diameter of the bar.

(vi) The anchorage value of a standard  $90^\circ$  bend is 12 times of the diameter of the bar.

Check for development length :-

$$\frac{M_1}{V} + L_0 \geq L_d$$

where,

$V$  = factored load

$M_1$  = Moment of resistance

$L_0$  =

If the check is not satisfied then following measure may be ~~adopted~~ adopted for satisfying the bend.

- 1) By reducing the diameter ( $\phi$ ) of the main steel to have lesser value of  $L_d$ .
- 2) By increasing the value of  $L_0$  by providing extra length of the bar over the bend.
- 3) By reducing number of bent up bars.

## Problem

An R.C.C. beam  $250\text{ mm} \times 500\text{ mm}$  has a clear span of  $5.5\text{ m}$ . The beam has 2- $20\text{ mm } \phi$  bars going into the support. Factored shear force is  $140\text{ kN}$ . Check for development length if Fe415 and M20 grade of concrete is used.

## Solution

Given,

$$b = 250\text{ mm}$$

$$D = 500\text{ mm}, \quad \phi = 20\text{ mm}, \quad n = 2$$

$$d = 500 - 20 - 10 = 470\text{ mm} \quad [\text{Assuming } 20\text{ mm Clear Covers}]$$

$$l = 5.5\text{ m}$$

$$f_{ck} = 20$$

$$f_y = 415$$

Act at support

$$= A \times \frac{\pi}{4} \times d^2$$

$$= 2 \times \frac{\pi}{4} \times 20^2$$

$$= 628.32\text{ mm}^2$$

$$V = 140\text{ kN} = 140 \times 10^3\text{ N}$$

Moment of resistance ( $M_1$ ) at the section

$$M_1 = 0.87 f_y A_{st} d \left( 1 - \frac{A_{st} f_y}{b d f_{ck}} \right)$$

$$= 0.87 f_y A_{st} d \left( 1 - \frac{A_{st} f_y}{b d f_{ck}} \right)$$

$$= 0.87 \times 415 \times 628.32 \times 470 \left( 1 - \frac{470 \times 415}{250 \times 470 \times 20} \right)$$

$$= 94.748 \times 10^6\text{ N mm}$$

Check for development length:

$$\frac{M_1}{V} + l_0 \geq L_d$$

$$L_d = \frac{0.87 f_y \phi}{4 \sigma_{bd}}$$

$$\sigma_{bd} = 1.6 \times 1.2 = 1.92 \text{ N/mm}^2 \text{ [for M20 concrete]}$$

$$L_d = \frac{0.87 \times 415 \times 20}{4 \times 1.92} = 940.2 \text{ mm}$$

\* Providing a  $90^\circ$  bend at the centre of support.

$$\text{Anchorage length, } l_0 = 8\phi = 8 \times 20 = 160 \text{ mm}$$

$$\frac{M_1}{V} + l_0 = \frac{94.748 \times 10^6}{140 \times 10^3} + 160$$

$$= 836.77 < L_d$$

Hence, there is a need to increase anchorage length, as codal requirements are not satisfied.

\* Providing U-bend at the end of the bar.

$$\text{Anchorage length} = 16\phi = 16 \times 20 = 320 \text{ mm}$$

$$\frac{M_1}{V} + l_0 = \frac{94.748 \times 10^6}{140 \times 10^3} + 320 = 997 \text{ mm} > L_d$$

$$\frac{M_1}{V} + l_0 > L_d$$

$\therefore$  Hence codal requirements are satisfied.

# New Chapter - 5

## T-Beams Analysis & Design of

T-Beams  
Written by: Asst. Prof. Satyajit Das, Dept. of CE

- The portion of the slab which acts integrally with the beam to resist loads is called as flange of the T-beam or L-beam.
- The portion of the beam below the flange is called as web or Rib of the beam.
- The intermediate beams supporting the slab are called as T-beams and the end beams are called as L-beams.

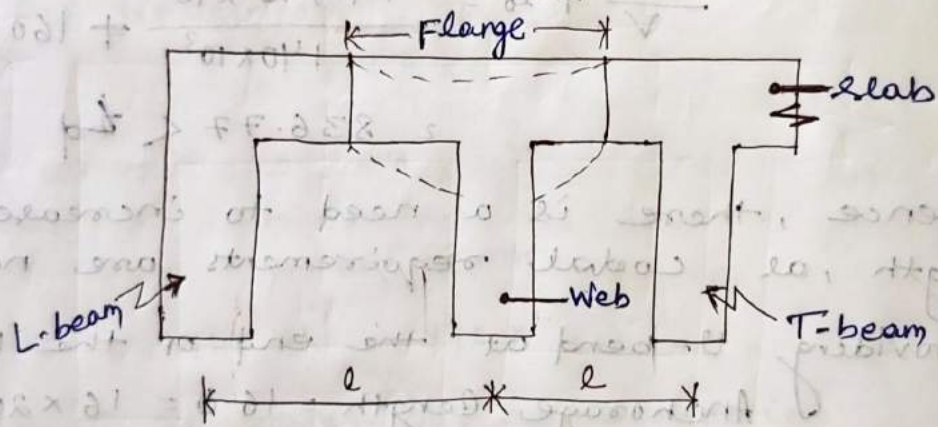


Fig. 1.1 - (T-beams & L-beams)

N.B

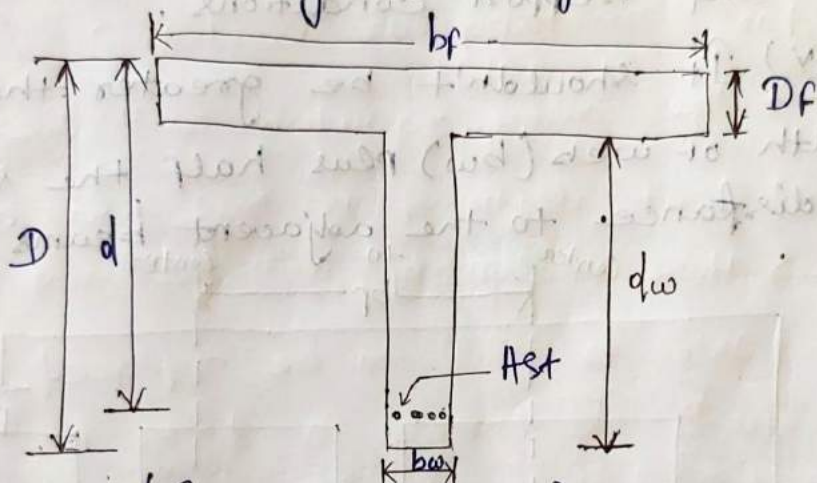
The flange of the beam (Part of the slab) contributes in resisting compression by adding more area of concrete in compression zone. This results in increasing moment of resistance of the beam section. However, if the flange is located in tension zone, the concrete area of the flange is to be neglected (cracked) and beam is treated as a rectangular beam.

### Breadth of web ( $b_w$ ):

Breadth of web is the width of the beam supporting the slab. The ratio of width of web to depth of web is kept as  $1/3$  to  $2/3$ .

### Thickness of the Flange ( $D_f$ ):

It is equal to the thickness or depth of the slab forming the flange of the beam.



(Fig 1.2 - T-beam)

### Overall depth of the beam ( $D$ ):

(i) It is equal to the sum of the depth of flange ( $D_f$ ) and depth of the web ( $d_w$ ).

(ii) In case of simply supported beams, it is assumed as  $1/12$  to  $1/15$  of the span.

(iii) for continuous beam, the overall depth is assumed as follows.

(a) for light loads:  $1/15$  to  $1/20$  of span (length)

(b) for medium loads:  $1/12$  to  $1/15$  of span

(c) for heavy loads:  $1/10$  to  $1/12$  of span

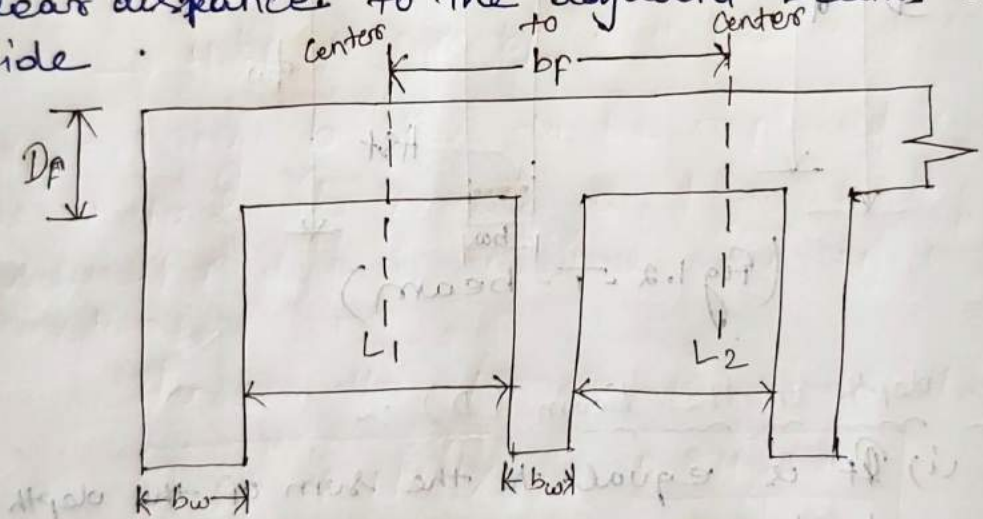
Effective width of the flange ( $b_f$ ):

(i) It is that portion of slab which acts integrally with the beam and extends on either side of the beam forming the compression zone.

(ii) Broadly, it depends upon the span of beam, thickness of slab and the breadth of the web.

(iii) Also it depends upon the type of loads and support conditions.

(iv) It shouldn't be greater than the breadth of web ( $b_w$ ) plus half the sum of clear distances to the adjacent beams on either side.



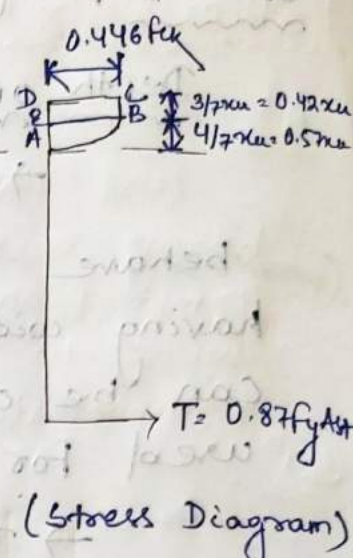
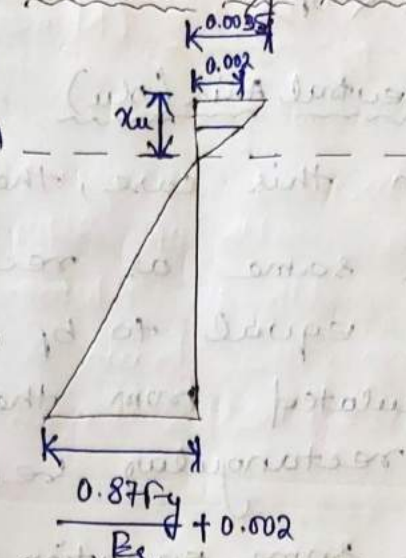
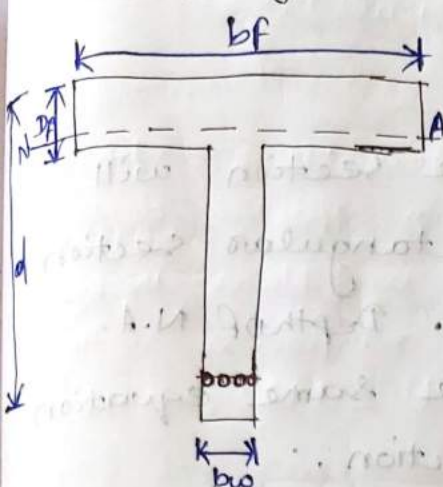
( Fig. 1.3 - Effective width of flange )

The effective width of compression flange of the flanged beam can be calculated as follows:

→ For T-beams:  $b_f = \frac{l_0}{6} + b_w + 6D_f$

→ For L-beams:  $b_f = \frac{l_0}{12} + b_w + 3D_f$

# Analysis of T-beams by LSM



(Section)

(Strain Diagram)

(Stress Diagram)

(Fig. 1.4 - Analysis of T-beam)  
( $x_u$ ,  $D_f$ )

## Depth of Neutral Axis and Moment of Resistance

Basically, there are two possible cases:

- (i) Neutral axis falls in the flange (i.e.  $x_u < D_f$ )
- (ii) Neutral axis falls in the web (i.e.  $x_u > D_f$ )

It looks alike Previous findings for singly reinforced beam analysis. Don't be confuse. Here  $D_f$  is a new term served with  $x_u$ , only for T-beams.

for students only

- (i)  $x_u < D_f$  (flange)
- (ii)  $x_u > D_f$  (web)
- (iii)  $x_u > D_f$  (web)



Case - 1 :- ( $x_u < D_f$ )

Depth of Neutral Axis ( $x_u$ )

→ In this case, the section will behave as same as rectangular section having width equal to  $b_f$ . Depth of N.A. can be calculated from the same equation used for rectangular section.

→ The same equation will be used to find out depth of neutral axis as before.

→ Here, only  $b_f$  will be used instead of  $b$ . Only ' $b$ ' has to be replaced with ' $b_f$ ' as before the formula used to find out the value for depth of neutral axis of a rectangular section.

Hence, the formula is given below:

$$\text{Depth of N.A. } (x_u) = \frac{0.87 f_y A_s}{0.36 f_{ck} b_f}$$

Moment of Resistance ( $M_u$ )

By comparing the values of  $x_u$  and  $x_{u,max}$ , the type of section can be determined as follows.

- (i)  $x_u < x_{u,max}$  (Under-reinforced section)
- (ii)  $x_u = x_{u,max}$  (Balanced section)
- (iii)  $x_u > x_{u,max}$  (Over-reinforced section)

# Ways to find out M.O.R. values for different type of sections

(i) In case of Under-reinforced section ( $x_u < x_{u\max}$ ),  
M.O.R. can be calculated as  
$$M_u = 0.87 f_y A_{st} (d - 0.42 x_u)$$

(ii) In case of Balanced section ( $x_u = x_{u\max}$ ),  
M.O.R. can be determined as  
$$M_{u\lim} = 0.36 f_{ck} b_f x_{u\max} (d - 0.42 x_{u\max})$$
  
or  
$$M_{u\lim} = 0.87 f_y A_{st} (d - 0.42 x_{u\max})$$

(iii) In case of Over-reinforced section ( $x_u > x_{u\max}$ )  
M.O.R. can not be fetched. Just mention,  
'it should be redesigned.'

Case-2 :- ( $x_u > D_f$ )

Not required <sup>for now</sup>. As, it is out of S.C.T.E & V.T  
syllabus. for details go to know about it  
go through text book of R.C.C.

Problem belongs to this chapters (Numericals):

Q.1

Find the moment of resistance of a T-beam having a web width of 240mm, effective depth of 400mm, flange width of 740mm and flange thickness equal to 100mm. The beam is reinforced with 5-16mm diameters, Fe 415 bars. Use M20 concrete.

Sol<sup>n</sup>

(Given)

$$b_w = 240 \text{ mm}$$

$$d = 400 \text{ mm}$$

$$b_f = 740 \text{ mm}$$

$$D_f = 100 \text{ mm}$$

$$n = 5$$

$$\phi = 16 \text{ mm}, f_{ck} = 20 \text{ N/mm}^2, f_y = 415 \text{ N/mm}^2$$

$$A_{st} = \frac{n \pi}{4} \times \phi^2$$

$$= 5 \times \frac{\pi}{4} \times 16^2$$

$$= 1005.3 \text{ mm}^2$$

→ Assuming the neutral axis to fall in the flange

$$x_u = \frac{0.87 f_y A_{st}}{0.36 f_{ck} b_f} = \frac{0.87 \times 415 \times 1005.3}{0.36 \times 20 \times 740}$$

$$x_u = 68.1 \text{ mm}$$

By Comparing, the value of  $x_u$  &  $D_f$ ,  
 $x_u < D_f$ .

Hence, Neutral axis lies in the flange.

$$x_{u\max} = 0.48d = 0.48 \times 400 = 192 \text{ mm}$$

By comparing the values of  $x_u$  and  $x_{u\max}$

$$x_u < x_{u\max}$$

Hence, the section is under-reinforced.

Therefore, Moment of resistance will be

$$M_u = 0.87 f_y A_{st} (d - 0.42 x_u)$$

$$= 0.87 \times 415 \times 1005 \cdot 3 (400 - 0.42 \times 68 \cdot 1)$$

$$= 134803942 \cdot 1 \text{ Nmm}$$

$$= 134.8 \text{ kNm}$$

$$\therefore M_u = 134.8 \text{ kNm}$$

(Ans)

Q.2

A T-beam floor system has 120 mm thick slab supported on beams. The width of beam is 300 mm and effective depth is 580 mm. The beam is reinforced with 8 bars of 20 mm diameter. Use M20 grade of concrete and Fe415 steel. The beams are spaced 3 m centre to centre. The effective span of beam is 3.6 m.

Given,  $b_w = 300 \text{ mm}$

$$d = 580 \text{ mm}$$

$$n = 8$$

$$\phi = 20$$

$$A_{st} = n \times \frac{\pi}{4} \times \phi^2 = 8 \times \frac{\pi}{4} \times 20^2$$

$$= 2513 \text{ mm}^2$$

$$D_f = 120 \text{ mm}, L = 3.6 \text{ m} = 3600 \text{ mm}$$

# Hints

Here effective width of flange i.e.  $b_f$  is unknown. the value of  $b_f$  has to be determined.

Effective width of flange ( $b_f$ )

$b_f$  can be determined as follows

$$b_f = \frac{l_0}{6} + b_w + 6 D_f \quad [\text{Simply supported beam}]$$

$$l_0 = L = 3600 \text{ mm} \quad (\because L = 3.6 \text{ m} = 3600 \text{ mm})$$

$$b_f = \frac{3600}{6} + 300 + 6 \times 120$$

$$= 1620 \text{ mm}$$

Depth of neutral axis ( $x_u$ )

Assuming neutral axis to fall in the flange

$$x_u = \frac{0.87 f_y A_{st}}{0.36 f_{ck} b_f} = \frac{0.87 \times 415 \times 2513}{0.36 \times 20 \times 1620}$$

$$x_u = 77.8 \text{ mm}$$

By comparing  $x_u$  &  $D_f$ ,

$$x_u < D_f$$

Hence, neutral axis lies in the flange.

$$x_{u \max} = 0.48 d$$

$$= 0.48 \times 580$$

$$= 278.4 \text{ mm}$$

By comparing the values of  $x_u$  and  $x_{u \max}$ ,

$$x_u < x_{u \max}$$

Hence, the section is under-reinforced.

Moment Of Resistance ( $M_u$ )

Hence, M.O.R. will be

$$M_u = 0.87 f_y A_{st} (d - 0.42 x_u)$$

$$= 0.87 \times 415 \times 2513 (580 - 0.42 \times 77.8)$$

$$= 496597272.7 \text{ Nmm}$$

$$M_u = 496.59 \approx 496.6 \text{ kNm}$$

$$\therefore M_u = 496.6 \text{ kNm} \quad (\underline{\text{Ans}})$$

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New  
Chapters-6

Analysis and Design

slab and stair case (LSM)

Turn Next Page  $\rightarrow$



New chapters - 6

# Analysis and Design of

## Slab and Stairs Case (LSM)

Written by Satyajit Das

Intro  
(i) Slab is a 2-D or planar element, used in all types of structure such as floors and roof coverings. The thickness of slab is very small as compared to its length and width.

(ii) Slabs are classified on the basis of  $\frac{l_y}{l_x}$  ratio, i.e. length of the longer span ( $l_y$ ) / length of the shorter span ( $l_x$ ) into following types.

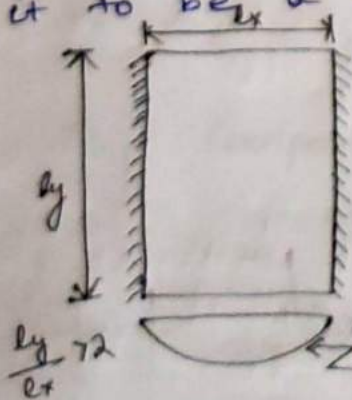
- (i) One way slab
- (ii) Two way slab

### One way slab :-

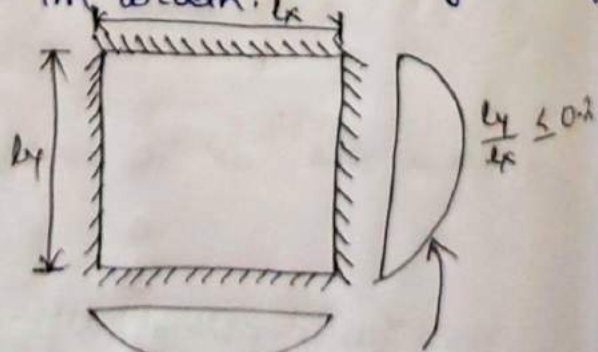
(i) One way slabs are those slabs in which the  $l_y/l_x$  ratio is greater than 2.

(ii) This type of slab is also called as slab spanning in one direction as the bending takes place only along the shorter span. So that, main reinforcement is provided along the shorter span.

(iii) The one way slab is analysed by assuming it to be a beam of 1m width,  $l_x$



Bending in shorter direction  
[ One-way slab ]



Bending takes place along both directions

## Two way slab :-

(i) The slab which is supported on all the four edges and having  $l_y/l_x$  ratio as less than 2 is called as two way slab.

(ii) Two way slab is also called as slab spanning in two directions because bending takes place in both the directions.

(iii) Main reinforcement is provided along both the directions in a two way slab.

## Deflection Control :-

For slabs, the vertical deflection limits are

provided by maximum  $l/d$  ratio:

(i) For span up to 10m	$l/d$ ratio
Simply supported	$\frac{20}{7}$
Cantilever	
Continuous	26

(ii) For span greater than 10m, the above mentioned value may be multiplied by  $10/\text{span}$ , except for cantilevers, for which exact deflection calculations should be made.

## Design of One way slab :-

One way slab is designed exactly as a rectangular beam, only difference are as given below.

(i) The width of the beam is assumed as one metre.

(ii) The depth of slab can be assumed on the basis of control of deflection. Using balanced Percentage of steel the  $(l/d)$  ratios are modified.



(iii) To start with the span to depth ratios are approximated as following for initial depth trial calculations.

→ for simply supported slabs, 25 to 30.

→ for cantilever slabs 10.

(iv) In addition to the main tensile reinforcement provided along shorter span, transverse reinforcement or distribution reinforcement is provided.

(v) Some of the main bars in a slab are bent up near the supports ( $\frac{l}{7}$  from centre of the support).

(vi) Shear is to be checked only. No shear reinforcement is provided.

Problem

Design a simply supported roof slab for a room  $7.5\text{m} \times 3.5\text{m}$  clear in size. The slab is carrying an imposed load of  $5\text{ kN/m}^2$ . Use M20 mix and Fe 415 steel.

Sol<sup>n</sup>

$$l_y/l_x = \frac{7.5}{3.5} > 2,$$

hence, it is a one way slab.

$$\text{Assuming total depth} = 150\text{ mm} \left[ d \approx \frac{l}{25} = \frac{3500}{25} = 140\text{ mm} \right]$$

$$d = 150 - 25 = 125\text{ mm} \quad [\text{clear cover } 20\text{ mm}]$$

and dia of main bar

effective span ( $l$ )

It should be least of the following:

(i) centre to centre distance =  $3.5 + 0.2 = 3.7\text{m}$

[assuming bearing length]

(ii) Clear span + effective depth =  $3.5 + 0.125 = 3.625 \text{ m}$   
 $l = 3.625 \text{ m}$

Factored load, Effective depth required, Area of tensile steel will be calculated using the same formulae used for rectangular section.

Self wt. of slab =  $0.15 \times 1.0 \times 25$  [unit wt. of R.C.C =  $25 \text{ kN/m}^3$ ]  
 $= 3.750 \text{ kN/m}$

Check for shear: Total load ( $w$ ) =  $5 + 3.75 = 8.75 \text{ kN/m}$   
 Design load ( $w_u$ ) =  $8.75 \times 1.5 = 13.125 \text{ kN/m}$

Factored shear force,  $V_u = \frac{w_u \cdot l}{2}$   
 $= \frac{13.125 \times 3.5}{2}$   
 $= 22.97 \text{ kN}$   
 $= 22970 \text{ N}$

Nominal shear stress,  $\tau_v$

$\tau_v = \frac{V_u}{bd} = \frac{22970}{1000 \times 125} = 0.18 \text{ N/mm}^2$

Design shear strength of concrete ( $\tau_c$ )

$P_t = \frac{100 A_{st}}{bd}$

' $\tau_c$ ' value will be calculated

then check & compare the values of  $\tau_v$  and

$\tau_c$

Check for deflection control

$$P_t = \frac{150 A_{st}}{bd}$$

Here, find out the values for  $P_t$ ,  $P_s$  &  $K_t$   
also find the values of  $(l/d)_{max}$  and  $(l/d)_{provided}$

Compare the values, if  $(l/d)_{max}$  is maximum than  $(l/d)_{provided}$ . Hence O.K.

Check for development length

It will be checked using  $\frac{M_1}{V} + l_o \leq l_d$  relation.

Cantilever slab or Chajja

One way cantilever slab or chajja is designed as a cantilever beam of one metre width. The steps to be followed in design of one way cantilever slab are given below.

(i) The effective span of cantilever slab is equal to the unsupported or projecting length of slab.

(ii) The effective depth at fixed end is maximum and is assumed to be about  $\frac{span}{10}$  to  $\frac{span}{12}$  from deflection consideration.

(iii) The depth required at the free end is minimum and is kept  $1/2$  to  $1/3$  of the depth at fixed end.

(iv) The main reinforcement is provided at the top and is to be curtailed at appropriate point.

For problems based on Cantilever Slab, follow Text book of R.C.C.